

## Propagation of Ion-acoustic Shocks in Electron-positron-ion Magnetoplasmas with Non-extensivity and Rotational Effects

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### ABSTRACT

Nonlinear electrostatic shock structures in dissipative magneto-rotating electron-positron-ion (e-p-i) plasmas with warm ions, non-thermal electrons and positrons following the  $q$ -nonextensive velocity distribution are investigated. The Korteweg de Vries Burger (KdVB) equation which describes the dynamics of the nonlinear shock structures is derived by using small amplitude reductive perturbation technique. The quantitative analysis of different physical parameters on the shock structures is presented here. It is found that the shock structures are sensitive to the Coriolis force, obliqueness, entropic indices of electrons and positrons ( $q_e$  and  $q_p$ ), ions temperature, positrons temperature and to the positrons concentration. This study would be helpful to understand the dynamics of the shock structures in the subextensive and superextensive plasmas with warm ions such as astrophysical and space environment.

### 1. Introduction

The ion acoustic wave (IAW) is a low frequency electrostatic mode which involves plasma density perturbations [1, 2]. The small as well as large amplitude IAW leads to different types of nonlinear structures such as solitons, shocks and vortices. It is well known that in a nonlinear dispersive media the solitary structures formed due to balance between the nonlinearity and dispersion which also leads to the formation of the shocks in the presence of dissipation [3]. The nonlinear structures have been found in laboratory as well as in space plasmas. Some authors have presented the observations of Freja and Viking satellites in order to understand the existence of various nonlinear structures in the planetary magnetospheres [4]. For this reason theoretical study of the nonlinear structures in space and laboratory plasmas have gained particular interest amongst the physicists. Cairns et al., 1995 investigated the effect of superthermal particles on the electrostatic solitary structures. The results of their study are in good agreement with the satellite observations [5]. Numerous other studies have also proved that Boltzmann Gibbs statistics based on the Maxwellian distribution is not a valid description for space plasmas under all conditions. There are various physical factors that create non-Maxwellian particle velocity distributions. For example the spatial variations of plasma density, temperature and the magnetic field as well as the background turbulence lead to the non-Maxwellian behavior of the particles for which

Boltzmann Gibbs (BG) statistics is not valid description. The space plasmas such as the solar wind, the planetary magnetospheres and the interstellar medium are well known examples of non-equilibrium systems which contain significant population of superthermal particles. For description of such systems Kappa or Flat top distributions have been successfully applied [6]. Furthermore, the generalization of BG statistics based on Tsallis/ non-extensive entropy has been used for several decades. During the last few decades, it has been proved that the systems which present long-range interactions and long-time memory cannot be dictated by the conventional Boltzmann Gibbs statistics. The main reason is that the BG statistics is based on additive or extensive formalism. According to the additive property the entropy of the composite system (A+B) of two independent systems A and B is equal to the sum of the entropies of the subsystems A and B; i.e.,  $S^{(A+B)} = S^A + S^B$ . However, for the non-extensive system the entropy of the composition is:  $S_q^{(A+B)} = S_q^A + S_q^B + (1-q)S_q^A S_q^B$ , where  $q$  is the entropic index and it determines the deviation from the BG statistics. Tsallis entropy has been found in many systems e.g., long-range Hamiltonian and Gravitational systems such as dark matter holes, cosmic rays, galaxy clusters, pulsar magnetospheres and space plasmas [7]. The non-extensive approach is based on the  $q$  distribution for the particle velocities which can be written as:

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$$f_e(v) = C_q [1 - (q-1) \left( \frac{m_e v^2}{2k_B T_e} - \frac{e\Phi}{k_B T_e} \right)]^{\frac{1}{q-1}} \quad (1)$$

where  $C_q$  is the constant of normalization and  $q$  is the non-extensive parameter. In the limiting case when  $(q-1)$  the  $q$  velocity distribution reduces to the well known Maxwellian distribution. In the superextensivity limit:  $-1 < q < 1$ , the constant of normalization is

$$C_q = n_{e0} \frac{\Gamma(\frac{1}{1-q})}{\Gamma(\frac{1}{1-q} - \frac{1}{2})} \times \sqrt{\frac{m_e(1-q)}{2\pi k_B T_e}} \quad \text{and in subextensivity}$$

$$\text{limit: } q > 1, \quad C_q = n_{e0} \frac{1+q}{2} \frac{\Gamma(\frac{1}{1-q} + \frac{1}{2})}{\Gamma(\frac{1}{1-q})} \times \sqrt{\frac{m_e(1-q)}{2\pi k_B T_e}},$$

where  $\Gamma$  corresponds to the standard gamma function. In subextensivity limit the thermal cutoff on the maximum value of the electrons velocity is given as:

$$v_{max} = \sqrt{\frac{2k_B T_e}{m_e} \left( \frac{e\phi}{k_B T_e} + \frac{1}{q-1} \right)} \quad [9].$$

In this study we deal with the plasma system which is composed of electrons, positrons and ions also known as electron-positron-ion (e-p-i) plasma. The e-p-i plasmas found in several astrophysical environments such as in active galactic nuclei, in a pulsar magnetosphere, in neutron stars and in the early universe exhibit long-range interactions [10, 11]. It is well understood that the dominant constituents of these astrophysical plasmas such as the electrons and positrons can be described by the non-extensive velocity distributions. The  $q$  velocity distribution significantly affects the nonlinear structures and for this reason it has been used in a large number of theoretical investigations during past few years. Many authors studied the excitation of the nonlinear electrostatic structures in various plasmas [12]-20]. The purpose of the present study is to investigate the propagation properties of the shock structures in dissipative magneto-rotating e-p-i plasmas. The propagation characteristics of the small amplitude perturbations in the dissipative plasma can be adequately described by the non-linear Korteweg-de Vries Burgers (KdVB) equation. The dissipative Burger term in the KdVB equation comes from the kinematic viscosity of the inertial ion fluid. A small amplitude reductive perturbation scheme is used to derive the KdVB equation. The effects of various parameters such as the Coriolis force, obliquity, temperature, positron concentration and nonextensivity are also presented. It is found that the kinematic viscosity plays key role in the formation of the shock structures in magnetized, rotating and dissipative e-p-i plasma. The remainder of this article is organized in the following manner: Section II briefly explains the model and governing equations. Section III deals with the

mathematical approach which is used to derive Korteweg de Vries Burger (KdVB) Equation. In section IV, we present results and the effects of various plasma parameters. Finally, Section V contains a summary and a general discussion of our findings.

## 2. Model

We consider the propagation of ion acoustic wave (IAW) in magnetized, rotating, and dissipative plasma whose constituents are hot ions and non-Maxwellian electrons and positrons. The model is electrostatic in which the ambient magnetic field is along  $z$ -axis (i.e.  $\vec{B} = B_0 \hat{z}$ ). The plasma is rotating with frequency  $\vec{\Omega}$  about the axis of rotation which makes angle  $\theta$  with the magnetic axis as illustrated in Fig. 1.

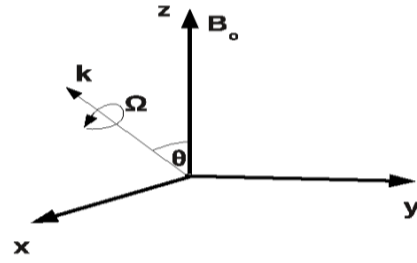


Fig. 1: Illustration of the geometry of the system

The rotational frequency  $\vec{\Omega}$  is given as  $\vec{\Omega} = (\Omega_0 \sin\theta, 0, \Omega_0 \cos\theta)$  with  $\Omega_0$  is the magnitude of the rotational frequency. The normalized continuity and momentum equations for the ion fluid are given as:

$$\frac{\partial n_i}{\partial t} + \nabla \cdot (n_i \vec{v}_i) = 0 \quad (2)$$

$$\frac{\partial \vec{v}_i}{\partial t} + (\vec{v}_i \cdot \nabla) \vec{v}_i = -\vec{\nabla} \phi - \vec{\nabla} \cdot \vec{P}_i + \Omega_{ci} (\vec{v}_i \times \hat{z}) + \frac{\mu_i}{m_i n_i} \nabla^2 v_i + 2(\vec{v}_i \times \vec{\Omega}) + \vec{\Omega} \times (\vec{\Omega} \times \vec{r}) \quad (3)$$

Where  $n_i$ ,  $m_i$  and  $\vec{v}_i$  represent ion density, mass and velocity with  $\Omega_{ci} = \frac{eB_0}{m_i}$  being the ion cyclotron frequency. Here  $\phi$  is the electrostatic potential,  $\vec{P}_i$  is the anisotropic pressure tensor and  $\mu_i$  is the dynamic viscosity of ions. The kinematic viscosity of the ion fluid is given as  $\mu_i = \frac{2.21 \times 10^{-15} T_i A_i}{Z^4 \ln \Lambda}$ , where  $A_i$  and  $Z$  represents the atomic weight and charge,  $T_i$  is the ion temperature and  $\ln \Lambda$  is the Coloumb logarithm. In Eq. 3

the last two terms represent the Coriolis force and Centrifugal force. We consider slow rotation; i.e.,  $\Omega_o < 1$  and therefore, the higher order terms of the rotational frequency  $\Omega_o$  are less significant. It can be seen that the Centrifugal force is directly proportional to  $\Omega_o$  and in slow rotation limit its effect is weak and can be neglected [18]. When plasma rotates its physical parameters exhibit their dependence on all three coordinates  $(x, y, z)$ ; i.e.,  $\vec{\nabla} = (\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z})$ . With these assumptions we found the following set of partial differential equations:

$$\frac{\partial n_i}{\partial t} + \frac{\partial(n_i v_{ix})}{\partial x} + \frac{\partial(n_i v_{iy})}{\partial y} + \frac{\partial(n_i v_{iz})}{\partial z} = 0, \quad (4)$$

The external magnetic field introduces anisotropy in the ions pressure which splits the pressure tensor into two components in the directions perpendicular and parallel to the magnetic field. The anisotropic pressure is modeled by using double adiabatic or Chew- Golberger-Low (CGL) theory which is given by the following relation [21- 24] :

$$\tilde{P}_i = p_{\perp i} \hat{I} + (p_{\parallel i} - p_{\perp i}) \hat{z} \hat{z}, \quad (5)$$

where  $\hat{I}$  is the unit tensor, and  $\hat{z}$  is the unit vector along the external magnetic field. The parallel and perpendicular components of the pressure are given as:

$$p_{\perp i} = p_{\perp io} \left( \frac{n_i}{n_{io}} \right), \quad (6)$$

$$p_{\parallel i} = p_{\parallel io} \left( \frac{n_i}{n_{io}} \right)^3, \quad (7)$$

here  $p_{\perp i} = p_{\perp io} \left( \frac{n_i}{n_{io}} \right)$  and  $p_{\parallel io} = n_{io} T_{\parallel i}$  represent the equilibrium ion pressures in perpendicular and parallel directions respectively. In isotropic case  $p_{\perp i} = p_{\parallel i}$ , and it leads to the following relation:  $\vec{\nabla} \cdot \tilde{P}_i = \vec{\nabla} p_i$ . The difference in the parallel and perpendicular temperatures also introduces the anisotropy in the kinematic viscosity of the ion fluid. The  $x, y,$  and  $z$  components of the momentum equation can be written as:

$$\begin{aligned} \frac{\partial v_{ix}}{\partial t} + (v_{ix} \frac{\partial}{\partial x} + v_{iy} \frac{\partial}{\partial y} + v_{iz} \frac{\partial}{\partial z}) v_{ix} = & -\sigma_{\perp i} \frac{1}{n_i} \frac{\partial n_i}{\partial x} \\ & - \frac{\partial \Phi}{\partial x} + (\omega_{ci} + 2\Omega_o \cos \theta) v_{iy} + \eta_{i\perp} \nabla^2 v_{ix} \end{aligned} \quad (8)$$

$$\begin{aligned} \frac{\partial v_{iy}}{\partial t} + (v_{ix} \frac{\partial}{\partial x} + v_{iy} \frac{\partial}{\partial y} + v_{iz} \frac{\partial}{\partial z}) v_{iy} = & -\sigma_{\perp i} \frac{1}{n_i} \frac{\partial n_i}{\partial y} - \frac{\partial \Phi}{\partial y} \\ & - (\omega_{ci} + 2\Omega_o \cos \theta) v_{ix} + 2\Omega_o \sin \theta v_{iz} + \eta_{i\perp} \nabla^2 v_{iy} \end{aligned} \quad (9)$$

$$\begin{aligned} \frac{\partial v_{iz}}{\partial t} + (v_{ix} \frac{\partial}{\partial x} + v_{iy} \frac{\partial}{\partial y} + v_{iz} \frac{\partial}{\partial z}) v_{iz} = & -3\sigma_{\parallel i} \\ & n_i \frac{\partial n_i}{\partial z} - \frac{\partial \Phi}{\partial z} - 2\Omega_o \sin \theta v_{iy} + \eta_{\parallel i} \nabla^2 v_{iz} \end{aligned} \quad (10)$$

In above equations  $v_{ix}$ ,  $v_{iy}$  and  $v_{iz}$  represent  $x, y$  and  $z$  components of the ions velocity,  $n_i$  is the ion density. The symbol  $\sigma_{\perp i}$  is the ratio between the perpendicular ion temperature ( $T_{\perp i}$ ) and electron temperature ( $T_e$ ); i.e.,  $\sigma_{\perp i} = \frac{T_{\perp i}}{T_e}$  and  $\sigma_{\parallel i}$  is the ratio between the parallel ion temperature ( $T_{\parallel i}$ ) and electron temperature ( $T_e$ ); i.e.,  $\sigma_{\parallel i} = \frac{T_{\parallel i}}{T_e}$ . The inertia less species such as electrons and positrons are described by the non-extensive  $q$  velocity distribution which in the normalized form can be written as :

$$n_p = v [1 - (q_p - 1) \sigma_p \Phi]^{-\frac{q_p + 1}{2(q_p - 1)}} \quad (11)$$

here  $q_e$  and  $q_p$  are the entropic indices for the electrons and positrons respectively. In above equations:  $\sigma_p = \frac{T_e}{T_p}$  denotes the ratio of electrons to positrons temperature,  $\mu = \frac{n_{eo}}{n_{io}}$  and  $\nu = \frac{n_{po}}{n_{io}}$ ,  $n_{eo}$ ,  $n_{po}$  and  $n_{io}$  represent unperturbed densities of electrons, positrons and ions respectively. At equilibrium the following quasineutrality condition holds:

$$\mu - \nu = 1 \quad (12)$$

Poisson's equation is given as:

$$\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} + \frac{\partial^2 \Phi}{\partial z^2} = n_e - n_p - n_i \quad (13)$$

Where  $\Phi$  is the normalized electrostatic potential. Following normalization is used:  $n_i$ ,  $n_e$  and  $n_p$  by  $n_{io}$ ,  $\vec{v}_i$  by ion acoustic speed  $C_s = \sqrt{\frac{k_B T_e}{m_i}}$ , the ion cyclotron frequency  $\Omega_{ci}$  and plasma rotational frequency

$\Omega_o$  are normalized by the ion plasma frequency

$\omega_{pi} = \sqrt{\frac{n_{io}e^2}{\epsilon_o m_i}}$  and  $\Phi$  by  $\frac{k_B T_e}{e}$ . The spatial and temporal scales are normalized by the Debye length ( $\lambda_D$ ) and  $\omega_{pi}^{-1}$  respectively,  $\eta_{i\perp} = \frac{\mu_{\perp i} \omega_{pi}}{C_s^2}$  and

$\eta_{\parallel i} = \frac{\mu_{\parallel i} \omega_{pi}}{C_s^2}$  are the normalized kinematic viscosities

of the ion fluid in the direction perpendicular and parallel to the magnetic axis.

### 3. Reductive Perturbation Scheme

A small amplitude reductive perturbation method is used to derive the KdVB equation [2]. The following stretching variables are used:

$$\xi = \varepsilon^{1/2}(l_x x + l_y y + l_z z - \lambda t), \tau = \varepsilon^{3/2} t \quad (14)$$

Here  $l_x$ ,  $l_y$  and  $l_z$  are the direction cosines of the wave vector in the direction perpendicular and parallel to the magnetic field,  $\lambda$  is the phase velocity of the ion acoustic wave (IAW) and  $\varepsilon$  is the small parameter ( $0 < \varepsilon < 1$ ) which determines the weakness of the nonlinearity. The physical quantities  $n_i$ ,  $v_{ix}$ ,  $v_{iy}$ ,  $v_{iz}$  and  $\Phi$  are expanded in terms of  $\varepsilon$ . The expansions used are given as:

$$n_i = 1 + \varepsilon n_i^{(1)} + \varepsilon^2 n_i^{(2)} + \dots \quad (15)$$

$$v_{ix} = \varepsilon^2 v_{ix}^{(1)} + \varepsilon^3 v_{ix}^{(2)} + \dots \quad (16)$$

$$v_{iy} = \varepsilon^2 v_{iy}^{(1)} + \varepsilon^2 v_{iy}^{(2)} + \dots \quad (17)$$

$$v_{iz} = \varepsilon v_{iz}^{(1)} + \varepsilon^2 v_{iz}^{(2)} + \dots \quad (18)$$

$$\Phi = \varepsilon \Phi^{(1)} + \varepsilon^2 \Phi^{(2)} + \dots \quad (19)$$

$$\eta_{\parallel i} = \varepsilon^{\frac{1}{2}} \eta_{\parallel o} \quad (20)$$

$$\eta_{\perp i} = \varepsilon^{\frac{1}{2}} \eta_{\perp o}. \quad (21)$$

Due to the anisotropy induced by the external magnetic field, the parallel component of the ion velocity  $v_{iz}$  is stronger than the perpendicular components  $v_{ix}$  and  $v_{iy}$ . This affect is noticeable from the powers of  $\varepsilon$  that is,  $v_{iz}$  contain lower powers of  $\varepsilon$  as compare to  $v_{ix}$

and  $v_{iy}$ . By transforming the above set of equations in new variables  $\zeta$ ,  $\tau$  and using the perturbation scheme given in Equations (16) to (22) we get different order terms of  $\varepsilon$ . The lowest order terms of  $\varepsilon$  give the following set of equations:

$$n_i^{(1)} = \frac{l_z}{\lambda} v_{iz}^{(1)} \quad (22)$$

$$v_{iy}^{(1)} = \frac{l_x}{(\omega_{ci} + 2\Omega_o \cos \theta)} (\partial_{\zeta} \Phi^{(1)} + \sigma_{\perp i} \partial_{\zeta} n_i^{(1)}) \quad (23)$$

$$v_{ix}^{(1)} = \frac{\lambda}{(\omega_{ci} + 2\Omega_o \cos \theta)} \partial_{\zeta} v_{iy}^{(1)} \quad (24)$$

$$\lambda \partial_{\zeta} v_{iz}^{(1)} = l_z \partial_{\zeta} \Phi^{(1)} + 3l_z \sigma_{\parallel i} l_z \partial_{\zeta} n_i^{(1)} \quad (25)$$

$$n_i^{(1)} = Q \Phi^{(1)} \quad (26)$$

where  $Q = (\mu \frac{q_e + 1}{2} + \nu \sigma_p \frac{q_p + 1}{2})$ . Equations (23) to (27) are used to obtain the phase speed of IAW which is given as:

$$\lambda = \sqrt{\frac{1}{Q} (l_z^2 + \frac{2\Omega_o \sin \theta l_x l_z}{\omega_{ci} + 2\Omega_o \cos \theta}) + (3l_z^2 \sigma_{\parallel i} + \frac{2\Omega_o \sin \theta l_x l_z \sigma_{\perp i}}{\omega_{ci} + 2\Omega_o \cos \theta})} \quad (27)$$

It can be seen from Equation (27) that the phase speed of the ion acoustic wave in e-p-i plasma depends on various plasma parameters e.g. plasma temperature through  $\sigma_{\perp i}$  and  $\sigma_{\parallel i}$ , the degree of nonextensivity through parameter  $Q$ , the obliqueness  $\theta$ , the ion gyrofrequency  $\omega_{ci}$  and the rotational frequency  $\Omega_o$ .

When  $\theta = 0^\circ$  that is the rotational axis is aligned with the magnetic axis then the rotational effects in the phase speed disappear. In limiting case, when plasma is composed of the nonextensive electron and cold ions  $\sigma_{\perp} = \sigma_{\parallel} = 0$  and  $q_p = 0$ , the phase speed reduces to:

$$\lambda_c = \sqrt{\frac{1}{Q} (l_z^2 + \frac{2\Omega_o \sin \theta l_x l_z}{\omega_{ci} + 2\Omega_o \cos \theta})}$$

which is determined by Hussain et al., 2013 [18]. Furthermore, for Maxwellian plasma; i.e.,  $q_e = 1$  and  $q_p = 0$ , the phase speed is identical to the one reported by Mushtaq, 2010 [25]. The higher order terms of  $\varepsilon$  leads to the following set of equations:

$$\partial_\tau n_i^{(1)} + I_x \partial_\zeta v_{ix}^{(1)} + I_z \partial_\zeta (n_i^{(1)} v_{iz}^{(1)}) \quad (28)$$

$$\lambda \partial_\zeta v_{ix}^{(1)} - \sigma_{\perp i} I_x n_i^{(1)} \partial_\zeta n_i^{(1)} = \sigma_{\perp i} I_x \partial_\zeta n_i^{(2)} + I_x \partial_\zeta \Phi^{(2)} - (\omega_{ci} + 2\Omega_o \cos \theta) v_{iy}^{(2)} \quad (29)$$

$$\lambda \partial_\zeta v_{iy}^{(1)} = (\omega_{ci} + 2\Omega_o \cos \theta) v_{ix}^{(1)} \quad (30)$$

$$\begin{aligned} \partial_\tau v_{iz}^{(1)} + I_z v_{iz}^{(1)} \partial_\zeta v_{iz}^{(1)} + 3\sigma_{\parallel i} I_z n_i^{(1)} \partial_\zeta n_i^{(1)} \\ - \eta_{\parallel o} \partial_\zeta v_{iz}^{(1)} = \lambda \partial_\zeta v_{iz}^{(2)} - I_z \partial_\zeta \Phi^{(2)} - \\ 2\Omega_o \sin \theta v_{iy}^{(2)} - 3\sigma_{\parallel i} I_z \partial_\zeta n_i^{(2)} \end{aligned} \quad (31)$$

$$\begin{aligned} \partial_{\zeta\zeta\zeta} \Phi^{(1)} - \frac{\mu(q_e + 1)(3 - q_e) - \nu\sigma_p^2(q_p + 1)(3 - q_p)}{4} \\ \Phi^{(1)} \partial_\zeta \Phi^{(1)} = Q \partial_\zeta \Phi^{(2)} - \partial_\zeta n_i^{(2)} \end{aligned} \quad (32)$$

By eliminating  $n_i^{(2)}$ ,  $v_{iy}^{(2)}$ ,  $v_{iz}^{(2)}$  and  $\Phi^{(2)}$  from Equations 29 -33, we get the KdVB equation for the ion acoustic shocks in magnetized rotating e-p-i plasma. The KdVB equation is given as:

$$\partial_\tau \Phi^{(1)} + A\Phi^{(1)} \partial_\zeta \Phi^{(1)} + B \partial_{\zeta\zeta\zeta} \Phi^{(1)} + C \partial_{\zeta\zeta} \Phi^{(1)} = 0 \quad (33)$$

Here  $A$ ,  $B$ , and  $C$  represent the nonlinear, dispersive and dissipative coefficients respectively. These coefficients can be given as:

$$\begin{aligned} A = \frac{1}{2\lambda Q} [Q^2 (3\lambda^2 + \sigma_{\parallel i} I_z - \frac{2\Omega_o \sin \theta I_x I_z \sigma_{\perp i}}{(\omega_{ci} + 2\Omega_o \cos \theta)}) \\ - \frac{(\lambda^2 - \lambda_c^2)}{4} (\mu(q_e + 1)(3 - q_e) - \nu\sigma_p^2(q_p + 1)(3 - q_p))] \end{aligned} \quad (34)$$

$$\begin{aligned} B = \frac{1}{2\lambda Q} [(\lambda^2 - \lambda_c^2) + \frac{\lambda^2 (1 + Q\sigma_{\perp i})}{(\omega_{ci} + 2\Omega_o \cos \theta)^2} \\ (I_x^2 - \frac{2\Omega_o \sin \theta I_x I_z \sigma_{\perp i}}{(\omega_{ci} + 2\Omega_o \cos \theta)})] \end{aligned} \quad (35)$$

$$C = \frac{\eta_{\parallel o}}{2} \quad (36)$$

#### 4. Results and Discussion

We have used the following analytical solution of the KdVB equation [3]:

$$\Phi = \Phi_o [ \sec h^2 (\frac{\zeta - \frac{6C^2}{25B} \tau}{\Delta}) + 2(1 - \tanh(\frac{\zeta - \frac{6C^2}{25B} \tau}{\Delta})) ] \quad (37)$$

where  $\Phi_o = \frac{3C^2}{25AB}$  and  $\Delta = \frac{10B}{C}$  represent the maximum amplitude and width of the shock structure. In this section we investigate the dependence of the ion acoustic shocks in a magnetized dissipative e-p-i plasmas on relevant plasma parameters such as nonthermal positron populations, Coriolis force, magnetic field, obliqueness, positron to electron temperature ratio ( $\sigma_p$ ), ion to electron temperature ratio ( $\sigma_{\parallel i}$ ), nonextensivity, and kinematic viscosity. The variations in the phase speed of the ion acoustic shock waves with nonextensive

parameters  $q_e$  and  $q_p$  are shown in Fig. 2. It can be seen that the phase speed of the ion acoustic shocks decreases with increasing the entropic index of the electrons  $q_e$ . However the decreasing trend in the phase speed is different for the sub-extensive  $q_e > 1$  and super-extensive  $q_e < 1$  regime. It is found that the phase speed decreases fast in the super-extensive regime and it decreases slowly in the sub-extensive regime. On the other hand, the phase speed of the ion acoustic shocks decreases linearly with increasing the nonextensive parameter of the positrons  $q_p$ . The phase speed also

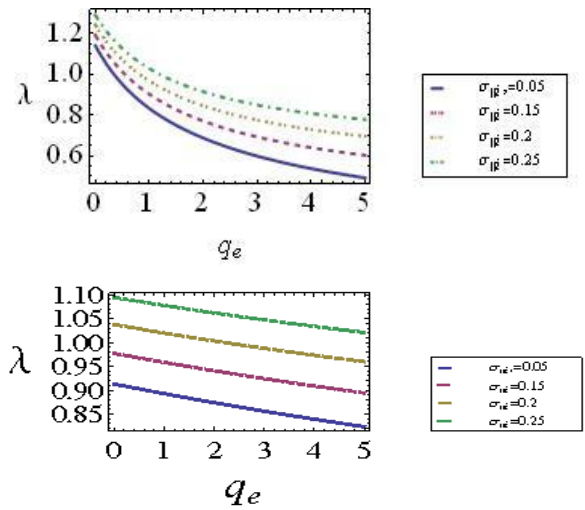


Fig. 2: Variation of the ion acoustic phase speed with parallel temperature

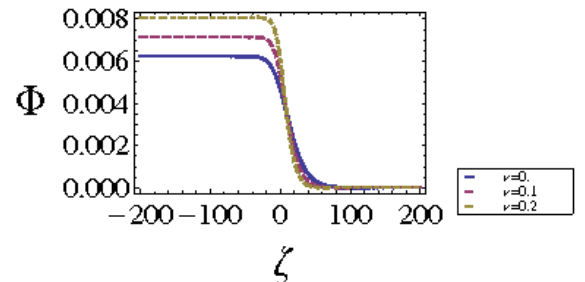


Fig. 3: Effect of the positron concentration on shock strength

exhibits strong dependence on the parallel component of the ion pressure or temperature and this effect is illustrated through  $\sigma_{//i}$  as shown in Fig. 2. It is found

that increasing  $T_{//i}$  through  $\sigma_{//i} = \frac{T_{//i}}{T_e}$  enhances the

phase speed of the ion acoustic shocks. Fig. 3 shows the impact of positron species on the propagation of the shocks in the system. It is noticed that by increasing the positron content ( $\nu$ ) the amplitude of the shock increases and its width decreases. This is due to decrease in the values of the coefficients **A** and **B** which in turn affect the amplitude and width of the shocks. The rotational frequency ( $\Omega_o$ ) determines the strength of the Coriolis force and also modifies the behavior of the nonlinear structure as illustrated in Fig. 4. It is noticed that by increasing the rotational frequency ( $\Omega_o$ ) the

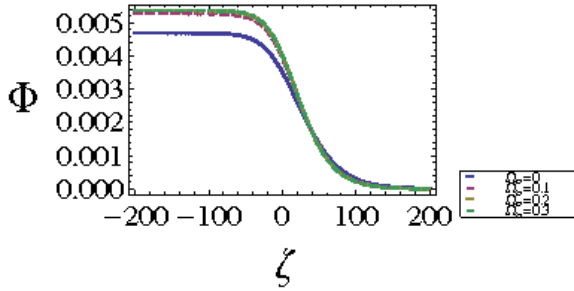


Fig 4: Effect of the rotational frequency on the strength of ion acoustic shocks

shock strength increases. The effect of the external magnetic field is studied through  $\omega_{ci} = \frac{\Omega_{ci}}{\omega_{pi}}$  as shown in

Fig. 5. It is noticed that by increasing the strength of the external magnetic field decreases the amplitude and increases the width of the shocks. We explore the dependence of the ion acoustic shock potential on the obliqueness through  $\theta$  which is the angle between wave propagation and the magnetic field as shown in Fig. 6. It is found that increasing the value of  $\theta$  enhances the amplitude of the ion acoustic shock as well as its steepness. The entropic indices  $q_e$  and  $q_p$  also affect

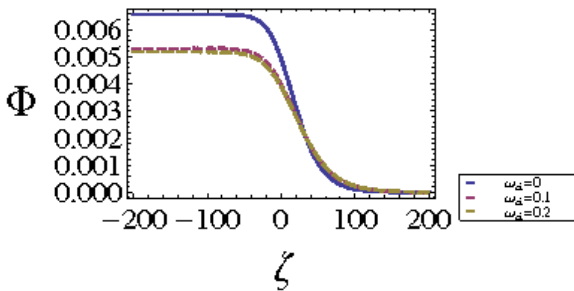


Fig 5: Effect of the magnetic field on the strength of ion acoustic shocks.

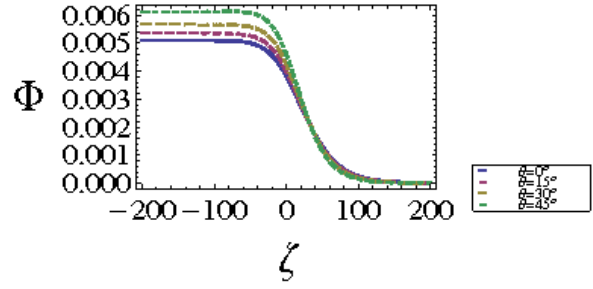


Fig 6: Effect of obliqueness on the strength of ion acoustic shocks

the properties of the ion acoustic shocks. In sub-extensive regime that is  $q_e = q_p > 1$ , the shock potential increases and width decreases with increasing the entropic parameters  $q_e$  and  $q_p$ . It is also observed that the shock amplitude increases and width decreases due to increase in the subthermality of the electrons and positrons. On the other hand it is noticed that increasing the superextensivity of electrons and positrons or decreasing the entropic parameter the shock strength decreases. The strength of the shocks strongly depends on the kinematic viscosity of the medium that is the shock strength increases with increasing  $\eta_{//o}$ . This is due to the fact that increasing the parallel kinematic viscosity is equivalent to high dissipation in the system which leads to a large value of the dissipation coefficient **C**. This in turn leads to large amplitude of the ion acoustic shock structures.

### 5. Conclusions

We have studied the propagation of ion acoustic (IA) shock waves in magnetized, rotating, dissipative, nonextensive e-p-i plasmas. The plasma is composed of inertial warm anisotropic ions and inertialess q-nonextensive distributed electrons and positrons. Small amplitude reductive perturbation technique is adapted to derive the Korteweg de Vries Burger (KdVB) equation which describes the dynamics of the ion acoustic shock structures in the system. The results show strong dependence on the plasma parameters which can be summarized here:

- The phase speed of the ion acoustic shocks shows strong dependence on the parallel component of the ion temperature. This in turn leads to the variation in the shocks potential at different ion temperatures.
- By increasing the positron concentration the ion acoustic shock strength increases.
- Increasing the strength of the Coriolis force enhances the ion acoustic shock potential.
- As the strength of the external magnetic field increases the shock potential decreases.
- The strength of the ion acoustic shock structure increases with increasing the subextensivity of the electrons and positrons. However, it decreases with

increasing superextensivity of the electrons and positrons. This is due to the fact that the entropic indices of the electrons and positrons affect the phase speed of the ion acoustic shocks reciprocally.

- Finally, the shock strength increases with increasing the dissipation coefficient in the system through the kinematic viscosity of the ion fluid.

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