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# Risk Estimation of Karachi Stock Exchange via Conditional Autoregressive Value-at-Risk by Regression Quantiles

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ABSTRACT

*This paper examines the market risk of Karachi Stock Exchange (KSE) by employing the Conditional Autoregressive Value-at-Risk by Regression Quantiles (CAViaR) model. The CAViaR model interprets the Value-at-Risk (VaR) as the quantile of future portfolio values conditional on current information and directly compute this quantile instead of inverting the distribution of returns. An asymmetric conditional heteroscedastic specification for CAViaR is proposed and applied along with four commonly used CAViaR specifications for the one-day-ahead VaR estimation of KSE for the period 1998 – 2010. The in-sample and out-of-sample predictive performance of alternative CAViaR specifications are compared and evaluated. The proposed model that accounts for asymmetry of risk is found to produce better and reliable estimates for VaR of KSE.*

## 1. Introduction

Financial institutions and their regulators use Value-at-Risk (VaR) as a standard tool to measure market risk. The need for developing practical and reliable risk management tools has been raised since the global financial crisis. The Basel Committee of Banking Supervision [1] has also called financial institutions such as banks for management of internal market risk. VaR is the quantile of the loss that can occur within a given portfolio during a specified time period. The estimation of VaR involves forecasting tail quantiles of the conditional return distribution. Thus, a precise quantile estimate far out in the left tail of the return distribution is desirable. A general introduction and exposition of VaR is provided by [2].

Several approaches of estimating VaR have been developed and substantial empirical application have emerged over the last two decades. Most of these methodologies are based on normality assumption of returns and mainly focused on the entire distribution of returns. A comprehensive overview and comparison of various approaches of estimating VaR can be found in [3] and [4].

An alternative approach that models the regression quantile instead of the entire distribution was proposed by [5]. The quantile regression method was first developed by [6] and a comprehensive review of development of these methods is provided in [7]. The model in [5] was called the Conditional Autoregressive Value-at-Risk by regression quantiles (CAViaR). This model provides an appealing approach for estimation of VaR and does not require the distributional assumptions as the quantiles are directly modelled using quantile regression. Through Monte Carlo simulation, they demonstrated that the CAViaR model outperformed other VaR approaches when return have fat tails. Empirical evidences of better predictive performance of CAViaR than the other VaR models are also reported in [8, 9].

Karachi Stock Exchange (KSE) is the major stock market of Pakistan. Few studies exist in literature that model the VaR of KSE. Parametric and non-parametric methods for computing VaR of KSE were used by [10]. Extreme value theory [11] and Historical Simulation and Risk Metrics methods [12] are also used in literature. The generalized autoregressive conditional heteroscedastic (GARCH) models have also been employed for risk estimation of KSE in [13, 14]. To the best of our knowledge, the CAViaR model which is shown to provide better VaR estimates has not been yet applied for risk estimation of KSE. This motivates us to employ this method to fill the gap and contribute to the existing literature of risk forecasting of KSE.

The main aims of this article is to apply CAViaR models of [5] for the estimation and prediction of VaR of KSE and propose an asymmetric GARCH specification of CAViaR model. The proposed asymmetric GARCH specification is based on the well-known model of [15]

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which is also known as the GJR model. Four different specifications, namely, Adaptive, Symmetric Absolute Value, Asymmetric Slope and Indirect GARCH (1,1) of [5] and the proposed Indirect GJR (1,1) are employed for the one-day-ahead VaR estimation of KSE in this article. The results are evaluated with various backtesting measures and tests. This study is important as KSE is the major stock market of Pakistan and reliable VaR estimates are desirable for the better risk management. The findings of this study may also help researchers, risk managers and practitioners in the country to choose consistent and reliable measures for risk during volatile period. Finally, it is shown through empirical application that the proposed CAViaR specification provides better VaR estimates than the competing models.

The rest of the paper is organized as follows. Section 2 discusses the CAViaR model and its four different specification. The proposed Indirect GJR(1,1) specification is also presented in this section . In Section 3, various specifications of the CAViaR model is applied to KSE data and results are presented and discussed. Finally, Section 4 summarizes the article and provides concluding comments.

## 2. Material and Method

The quantile regression method of [6] is used for the estimation of the parameters of CAViaR model. Let a sample of observations $y_1, \cdots, y_T$ are generated by the following model

$$y_t = x_t^{'}\beta_\theta + \epsilon_{\theta t}, \quad Quant_\theta(y_t|x_t) = x_t^{'}\beta_\theta \qquad (1)$$

where $x_t$ is a p-dimensional vector of regressors and $Quant_\theta(y_t|x_t) = x_t^{'}\beta_\theta$ is the $\theta$-quantile of $\epsilon_{\theta t}$ conditional on $x_t$ and $\beta_\theta$ is the vector of unknown parameters. For the sake of notational convenience, the subscript $\theta$ is eliminated from the vector of unknown parameters. Also define $f_t(\beta) = x_t^{'}\beta$, then the $\theta$th regression quantile is defined as any $\hat{\beta}$ that solves

$$\min_\beta \frac{1}{T}\sum_{t=1}^{T}\left[\theta - I\big(y_t < f_t(\beta)\big)\right][y_t - f_t(\beta)] \quad (2)$$

where $I(\cdot)$ is an indicator function. Hence quantiles can be defined through an optimization problem.

The CAViaR uses quantile regression and aims to directly model the required quantile of the return distribution rather than modelling the whole distribution of returns. Since the distribution of volatilities is auto-correlated over time, the model uses an autoregressive specification. Four different specification processes were proposed by [5] for the calculation of VaR. These are Adaptive, Symmetric Absolute Value, Asymmetric Slope and Indirect GARCH(1,1).

Following [5], consider an observable vector of portfolio returns, $\{y_t\}_{t=1}^{T}$ and define $f_t(\beta) \equiv f(x_{t-1}, \beta)$ to be the $\theta$-quantile of the distribution of the portfolio returns at time $t$ formed at time $t-1$. The first specification, an Adaptive model, is a smoothed version of a step function (for finite G>0), and takes the following form

$$f_t(\beta) = f_{t-1}(\beta_1) + \beta_1\{[1 + \exp(G[y_{t-1} - f_{t-1}(\beta_1)])]^{-1} - \theta\},$$

This model adapts itself depending on whether VaR is exceeded or not, it takes a higher value when VaR is exceeded but decreases slightly otherwise.

A second model is Symmetric Absolute Value model and is set out as

$$f_t(\beta) = \beta_1 + \beta_2 f_{t-1}(\beta) + \beta_3|y_{t-1}|$$

This model responds symmetrically to past portfolio returns and it is mean reverting since the coefficient of the lagged VaR is not constrained to equal one.

A third model is Asymmetric Slope model and is mathematically defined as

$$f_t(\beta) = \beta_1 + \beta_2 f_{t-1}(\beta) + \beta_3(y_{t-1})^+ + \beta_4(y_{t-1})^-,$$

where $(y)^+ = \max(y, 0)$, and $(y)^- = -\min(y, 0)$. This model allows for an asymmetric response to positive and negative past portfolio returns.

A fourth specification is Indirect GARCH(1,1) model having the following expression

$$f_t(\beta) = (\beta_1 + \beta_2 f_{t-1}^2(\beta) + \beta_3 y_{t-1}^2)^{1/2}$$

This specification can be correctly modelled under the assumption that the underlying data process follows a true GARCH(1,1) with an i.i.d. error distribution.

Finally, the proposed fifth specification is the Indirect GJR(1,1) model that can be defines using the following mathematical form:

$$f_t(\beta) = (\beta_1 + \beta_2 f_{t-1}^2(\beta) + \beta_3 y_{t-1}^2 + \beta_4\, y_{t-1}^2 I(y_{t-1} < 0))^{1/2}$$

where again $I(\cdot)$ is an indicator function. This specification has the same assumptions as Indirect GARCH(1,1) and besides allow the leverage effect to be estimated in VaR framework.

Some common criteria exists for the comparison of forecasting performance of VaR. Let us define the total number of observed violations as $T_*$

$$T_* = \sum_{t=1}^{T} I_t \quad \text{with} \quad I_t = I(y_t \le -\text{VaR}_t)$$

Then the closeness of empirical rejection probability $\hat{\theta} = T_*/T$ to '$\theta$' can be used to assess the overall predicative performance of the underlying VaR model. The value of $\hat{\theta}$ is called the violation rate (VRate). For accurate risk models, the VRate should be close to the risk level, $\theta$.

A Dynamic Quantile (DQ) test was proposed by [5] for the evaluation of alternative specifications of VaR models. The DQ test is used to check the high order dependence among $\{I_t\}$s. Define

$$Hit_t(\beta) \equiv I_t - \theta.$$

This function assumes the value $(1-\theta)$ every time a violation occurs and takes the value $-\theta$ otherwise. A regression type test is constructed that examines the dependence of $Hit_t$ to its own lagged values, lagged VaR forecasts and other regressors over time. Naturally, the model that produces accurate and independent $Hit_t$ should not be related to its lagged values and other regressors.

Two test statistics were derived by [5]. First, an in-sample DQ test is constructed. This test is used to select among alternative model specifications of a particular CAViaR process. Second, an out-of-sample DQ test is constructed which is useful to the market regulators and/or the risk managers, since they can examine whether VaR estimates satisfy certain properties such as unbiasedness, independence of hits and independent quantile.

In the next section, five CAViaR specifications outlined above is applied to the estimation of VaR for KSE. To assess the predictive performance of in-sample and out-sample VaR of competing models, the number of violations, VRate and DQ tests are used.

## 3. Results and Discussion

### *3.1 Data and Preliminary Analysis*

In the present study the daily closing prices of Karachi Stock Exchange (KSE 100 Index) are used. The dataset is obtained from the http://finance.yahoo.com for the period of January 05, 1998 to December 31, 2010. The specific period is used in this study as this period includes the high volatile period of the global financial crisis. This may be helpful to understand the market risk of KSE before and during financial crisis periods. The data set consists of 3167 observations.

The returns at time $t$ is defined as $r_t = \ln(P_t/P_{t-1}) \times 100\%$, for $t = 1, \dots, T$, where $P_t$ is the closing index of KSE at time $t$. Then using $\{y_t = r_t - \bar{r}_t; 1 \le t \le T\}$ (with $\bar{r}_t = \sum_{t=1}^{T} r_t/T$) as our observations, the whole span in each time period is divided into two parts: the estimation or in-sample part of initial $K$ observations used for estimating the unknown parameters in five CAViaR models and the validation or out-of-sample part of $N = T - K$ observations for the prediction and assessment of VaR. For out-of-sample forecasting, the sample of size $N = 500$ is chosen which corresponds to roughly two years of observations.

The daily log-returns of KSE are shown in Fig. 1. The effect of global financial crisis is evident on KSE. A high volatility and volatility clustering can also be seen in the log-returns series. Therefore, one of the main interests lies on the risk estimation during this period.

Summary statistics for KSE return series are presented in Table 1. The mean of the log-returns is close to 0. The standard deviation which is the historical volatility of KSE is 0.7531.The log-return series have higher kurtosis and negative skewness. Jarque-Bera test is used to assess the normality of returns and high value of this test indicates that KSE returns are significantly different from normal. Ljung-Box ($Q^2$) statistic for the squared returns at lag 20 were also found significant. This indicates the dependence in squared returns. To summarize, the KSE return series are non-normal, dependent on past observations and also have high kurtosis and asymmetry.

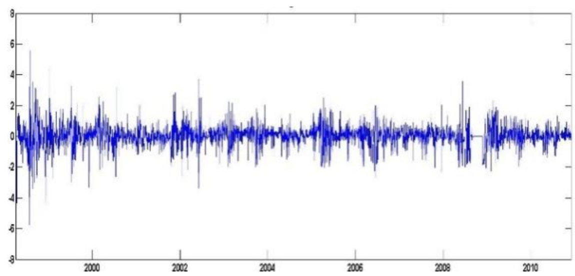


Fig. 1: Log-returns of Karachi Stock Exchange

Table 1: Summary statistics for daily return of Karachi Stock Exchange

| Sample size | 3167 |
|---|---|
| Mean | 0.02646 |
| Median | 0.0525 |
| Minimum | -5.7384 |
| Maximum | 5.5426 |
| SD | 0.7531 |
| Skewness | -0.3478 |
| Kurtosis | 8.1076 |
| JB | 3506.28 |
| $Q^2(20)$ | 1446.00 |

Note: JB (Jarque-Bera statistic for normality of return); $Q^2$ (Ljung-Box statistics at lag 20 for serial correlation in squared returns).

### *3.2 Value-at-Risk Estimation and Evaluation*

Both in-sample and out-sample VaR estimates are generated using alternative CAViaR specifications. In this study the focus is on 1% and 5% VaR levels. To produce one-day-ahead forecast of VaR, a rolling window approach is used. More specifically, the model is fitted to the estimation period using in-sample part of $K$ observations and one-day-ahead VaR forecasts are obtained. Then the in-sample period is rolled forward by one day dropping the first observation. The model is reestimated and again the next day forecasts are obtained. In this way out-sample VaR forecasts of approximately two years (500 days) are obtained for the forecast period. This will also allow each model to adapt to the varying risk dynamics and levels. The Matlab code was kindly provided by Simone Manganelli. The code was adapted an updated for Indirect GJR(1,1) specification.

Next, 1% and 5% one-day-ahead VaR is estimated using four specifications of CAViaR. For the Adaptive model, G is set to 10 as proposed [5]. The results for 1% and 5% VaR levels are summarized in Table 2 and 3, respectively. The top panel of each table shows the values of the estimated parameters of five CAViaR models, the respective standard errors and one sided p values. The regression objective function value (RQ) is reported in the bottom panel. Both in-sample and out-sample percentages of VaR violations and p values of DQ tests are also presented along with the $\mathrm{VRate}/\theta$. Ideally, the $\mathrm{VRate}/\theta$ should be close to 1 and for the sake of comparison, a value of 0.9 is considered better than 1.1 as the former value is considered conservative (see [16]).

Table 2: Estimates and Relevant Statistics for the four CAViaR Models at 1% VaR

| 1% VaR | SA | AS | IG | AD | I GJR |
|---|---|---|---|---|---|
| Beta 1 | 0.1447 | 0.2701 | 0.2751 | 0.5073 | 0.6180 |
| Standard errors | 0.0682 | 0.1167 | 0.1098 | 0.1376 | 0.1306 |
| p values | 0.0169 | 0.0103 | 0.0061 | 0.0001 | 0.0000 |
| Beta 2 | 0.8254 | 0.6772 | 0.8425 | | 0.5648 |
| Standard errors | 0.0660 | 0.0834 | 0.0341 | | 0.0313 |
| p values | 0.0000 | 0.0000 | 0.0000 | | 0.0000 |
| Beta 3 | 0.3341 | 0.2794 | 0.5449 | | 0.3931 |
| Standard errors | 0.1173 | 0.1015 | 0.3854 | | 0.2100 |
| p values | 0.0022 | 0.0029 | 0.0787 | | 0.0306 |
| Beta 4 | | 0.9115 | | | 2.7990 |
| Standard errors | | 0.1675 | | | 1.0278 |
| p values | | 0.0000 | | | 0.0032 |
| RQ | 55.6775 | 52.9780 | 55.2787 | 60.6229 | 52.5238 |
| Hits (%) (in) | 1.0152 | 1.0152 | 1.0614 | 0.6922 | 0.9691 |
| Hits (%) (out) | 1.2000 | 1.2000 | 1.0000 | 0.5000 | 1.1000 |
| VRate/θ (in) | 1.0152 | 1.0152 | 1.0614 | 0.6922 | 0.9691 |
| VRate/θ (out) | 1.2000 | 1.2000 | 1.0000 | 0.5000 | 1.1000 |
| DQ (in) | 0.1904 | 0.8760 | 0.3360 | 0.0002 | 0.9584 |
| DQ (out) | 0.0000 | 0.8191 | 0.0000 | 0.8568 | 0.9521 |

SAV: Symmetric Absolute Value; AS: Asymmetric Slope; IG: Indirect GARCH; AD: Adaptive; IGJR: Indirect GJR; RQ: Regression Obective Function Value; DQ: p-values of Dynamic Qunatile test

All the parameters are found statistically significant at 5% significance level except for $\beta_3$ in case of Indirect GARCH model which is found significant at 10% level of significance. The coefficient $\beta_2$ is found highly significant in both tables. This implies that volatility clustering is verified for the stock price returns of the KSE.

In order to check the accuracy of alternative models, we first focus our attention on the in-sample VaR. For the percentage of in-sample hits, it is observed that the all models except the Adaptive model at 1% and 5% VaR levels provide estimates that are reasonably close to value 1 and 5, respectively. This may be taken as evidence that these models describe the evolution of the tail for most of the cases. The $\mathrm{VRate}/\theta$ for in-sample is also found close to 1 for all models except the Adaptive model indicating that the risk in KSE is estimated accurately by these models in the period understudy.

Mixed results were observed for in-sample DQ test. At 1% VaR level, the DQ test was rejected for the Adaptive model at 5% significance level whereas for other models the non-significant values of this tests imply that VaR violations are independent. The Indirect GJR model provides the highest p value for this test. At 5% VaR level, the DQ test was found significant for Symmetric Absolute Value at 5% significance and Indirect GARCH at 10% significance level.

Table 3: Estimates and Relevant Statistics for the four CAViaR Models at 5% VaR

| 1% VaR | SA | AS | IG | AD | I GJR |
|---|---|---|---|---|---|
| Beta 1 | 0.0857 | 0.0933 | 0.0804 | 1.0434 | 0.0919 |
| Standard errors | 0.0238 | 0.0193 | 0.0292 | 0.0497 | 0.0241 |
| p values | 0.0002 | 0.0000 | 0.0029 | 0.0000 | 0.0000 |
| Beta 2 | 0.7601 | 0.7653 | 0.7354 | | 0.7034 |
| Standard errors | 0.0437 | 0.0244 | 0.0238 | | 0.0209 |
| p values | 0.0000 | 0.0000 | 0.0000 | | 0.0000 |
| Beta 3 | 0.3616 | 0.1149 | 0.5822 | | 0.2561 |
| Standard errors | 0.1017 | 0.0407 | 0.2355 | | 0.0621 |
| p values | 0.0002 | 0.0024 | 0.0067 | | 0.0478 |
| Beta 4 | | 0.5397 | | | 0.9930 |
| Standard errors | | 0.0675 | | | 0.2661 |
| p values | | 0.0000 | | | 0.0001 |
| RQ | 186.0600 | 178.2320 | 185.6386 | 192.7849 | 17.2737 |
| Hits (%) (in) | 5.0761 | 5.0300 | 5.0761 | 4.4762 | 5.0300 |
| Hits (%) (out) | 4.5000 | 4.1000 | 4.4000 | 3.7000 | 4.5000 |
| VRate/θ (in) | 1.0152 | 1.0060 | 1.0152 | 0.8952 | 1.0060 |
| VRate/θ (out) | 0.9000 | 0.8200 | 0.8800 | 0.7400 | 0.8800 |
| DQ (in) | 0.0042 | 0.1962 | 0.0664 | 0.7486 | 0.1180 |
| DQ (out) | 0.0018 | 0.4210 | 0.0455 | 0.0636 | 0.5090 |

SAV: Symmetric Absolute Value; AS: Asymmetric Slope; IG: Indirect GARCH; AD: Adaptive; IGJR: Indirect GJR; RQ: Regression Obective Function Value; DQ: p-values of Dynamic Qunatile test

Next, the out-sample VaR is considered. The percentage of out-sample hits are also found close to nominal values for these models at both 1% and 5% VaR levels. The Adaptive model produced the lowest percentage of out-sample hits as compare to other models. The $\mathrm{VRate}/\theta$ for out-sample of all models are found close to one at 1% VaR level except the Adaptive model which produces very low number of hits. At 5% VaR levels, the $\mathrm{VRate}/\theta$ are smaller than one for all models.

At 1% VaR level, the DQ test for out-sample was rejected in both Symmetric Absolute Value and Indirect GARCH models at 5% level of significance. The Indirect GJR model again provide the highest p value for this test. At 5% VaR level, the out-sample DQ test are found non-significant for Asymmetric Slope, and Indirect GJR models only at 10% level of significance with Indirect GJR model again producing the highest p value. The rejection of most of models for out-sample period are most likely due to the impact of the global financial crisis.

Fig. 2 shows the estimated 5% VaR of various specifications considered in this paper. The plot of Adaptive model is not presented as the performance of this model was found inferior then others. All conditional autoregressive VaR plots have shown to fluctuates with the volatility of KSE and provide reasonable risk estimates.

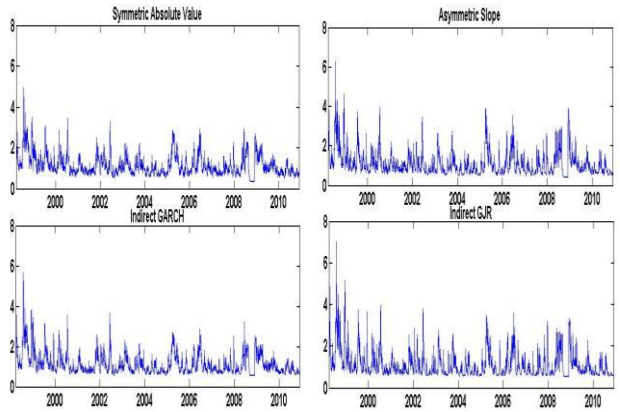


Fig. 2: Estimated conditional autoregressive VaR (5%) plot for KSE

To summarize, the Indirect GJR model proposed in this article performed reasonably well among other competing models for estimation and forecasting of one-day-ahead VaR. This model can capture the asymmetry in VaR and this may be one of the reason of the better performance. The KSE returns had slight skewness and this was reflected in the conditional risk as well. The proposed specification that incorporate skewness was found to provide a good fit to the conditional VaR of KSE.

## 4. Conclusions

Conditional autoregressive value-at-risk by regression quantiles (CAViaR) models are employed in this study to forecast the one-day ahead VaR of KSE for the period 1998 – 2010. This time period includes the global financial crisis and hence the study of risk dynamics for local market is important during this period of high and low volatility. Four CAViaR specifications, namely, Adaptive, Symmetric Absolute Value, Asymmetric Slope and Indirect GARCH are considered. Besides, an asymmetric specification of CAViaR called Indirect GJR that allows the leverage effect to be estimated in VaR framework is also proposed. Result of this study show that, in general, asymmetric specifications provide reasonable VaR estimates for KSE. The proposed CAViaR specification is found the best in terms of producing better VaR violations, VRate and non-significant DQ tests for both in-sample and out-sample, than the competing specifications. Hence, this study proposes the use of Indirect GJR specification for estimation and forecasting of risk of KSE. The implications of these results are of great importance particularly in case of financial risk management. The findings of this research may provide risk managers and practitioners an extra leverage to choose consistent and reliable measures for risk during volatile period. Moreover, these will also help financial institutions such as banks in Pakistan to consider and develop VaR models that incorporates asymmetry of risk for their internal risk management.

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