

Computing Shortest Path in a Single Valued Neutrosophic Hesitant Fuzzy Network

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ABSTRACT

In engineering, computer sciences and many other applied sciences, finding shortest path in a network is one of the famous problems. The aim of this manuscript is to develop a novel algorithm for finding shortest path in a network where nodes and edges have some uncertainty. Firstly, the concept of single-valued neutrosophic hesitant fuzzy graph (SVNHFG) has been introduced with some related graph theoretic results. Some examples are provided to understand the defined concepts. Then, the new algorithm for solving shortest path problems (SPPs) has been introduced followed by a flowchart for a stepwise description. A numerical example is provided in the environment of SVNHFGs to demonstrate the proposed algorithm. The advantages of proposed method over the existing techniques are also studied.

1. Introduction

The computation of shortest path is a basic problem in networks especially in computer, communication and some other networks in engineering sciences. In such problems, the target is to determine the shortest path between first and final vertex. These problems are discussed in computer sciences, graph theory and Applied Mathematics [1-3].

The SPP has also been discussed in fuzzy environment since the concept of fuzzy set (FS) has been introduced. In real-life problems, sometimes we face uncertainty in computation due to several unnoticeable reasons and FS is proved to be the best tool so far in handling such problems. SPP has been studied not only using fuzzy graphs (FGs) but also using intuitionistic fuzzy graphs (IFGs), neutrosophic graphs (NGs) especially single valued neutrosophic graphs (SVNGs) and various other generalized structures. For some significant work on shortest path in fuzzy environments, one may refer to previously reported literature [4-15].

The concept of FS was intimated by Zadeh [15] in 1965, which is proved to be a remarkable tool for handling uncertainties that exists in real life problems. This concept was further enhanced by Atanassov [16] and the concept of intuitionistic fuzzy sets (IFSs) have been proposed. The concept of IFS has its own limitations which further enhanced the concept of neutrosophic set (NS) by Smarandache [17], which leads to the concept of single valued neutrosophic set (SVNS) by Haibin et al. [18]. There are some other directions too where IFSs have been generalized such as the concept of picture fuzzy set (PFS) introduced by Cuong [19], spherical and T-spherical fuzzy sets by Mahmood et al. [20] and Ullah et al. [21]. The concept of hesitant fuzzy set (HFS) developed by Torra [22] has also a unique way of handling uncertainties in real life phenomena's. Combining different fuzzy algebraic structures to get a new structure is also common in FS theory

and several new directions have been reported in previously [23-28].

The theory of FGs was initiated by Kauffman [29] and further deeply studied by Rosenfeld [30]. FGs have been extensively studied and several new dimensions have been introduced. The idea of IFGs was developed by Parvathi and Karunambigai [31] and further investigated by Parvathi et al. [32-34] and Pasi et al. [35]. Similarly IFG was further enhanced to IFG of second type (IFGST) by Dhavudh and Srinivasan [36]. To make the domain of IFG more viable, the concept of IFGs of nth type (IFGNT) and complex intuitionistic fuzzy graphs (CIFG) are recently proposed by Davvaz et al. [37] and Yaqoob et al. [38]. Some study related to NGs and its generalizations is investigated recently [39-44]. Recently the study of hesitant fuzzy graphs (HFGs) is developed by Zhang and Li [45] where its several operations are defined and their applications in decision making are studied. The concept of cubic graphs is investigated by Rashid et al. [46].

Recently, the idea of single valued neutrosophic hesitant fuzzy set (SVNHFS) is proposed by Ye [47], which is a generalization of both SVNSs and HFSs. Motivated by the work on this recent development, in this article, we aimed to develop the concept of SVNHFGs and studied the famous SPP in a network which is based on SVNHFS information.

In this article, we discussed the significance of SPP in various fields especially in fuzzy algebraic structures in first section. The second section contained some basic definitions. Section three is based on some concepts of SVNHFGs and its related terms. The fourth section is based on a novel algorithm for SPP and its flowchart. In fifth section, we described a numerical example based on the proposed algorithm and demonstrated its every step with details. Finally, the article is summarized with some future directions are discussed.

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2. Preliminaries

This section describes some important terms which are helpful to study the paper. Here IFSs, HFSs and SVNNS are discussed and their graphs are demonstrated. Throughout our article, the terms $\hat{S}_1(\hat{S}_2)$, $\tilde{I}_1(\tilde{I}_2)$ and $\mathcal{D}_1(\mathcal{D}_2)$ will denote the membership, indeterminacy and non-membership functions of nodes (edges) respectively.

2.1 Definition [18]

An IFS is a duplet $\hat{A} = (\hat{S}, \mathcal{D})$ in set \tilde{X} where \hat{S}, \mathcal{D} are functions from the set \tilde{X} to an element from unit interval $[0,1]$ with a restriction that $0 \leq \hat{S} + \mathcal{D} \leq 1$.

2.2 Definition [34]

An IFG is a duplet $G = (\tilde{V}, \tilde{E})$ where \tilde{V} is a set of nodes s.t and \hat{S}_1, \mathcal{D}_1 are two mappings on $[0,1]$ interval for $\tilde{v}_n \in \tilde{V}$ respectively with a restriction $0 \leq \hat{S}_1 + \mathcal{D}_1 \leq 1$ and \tilde{E} is the set of edges where $e_i \in \tilde{V} \times \tilde{V}$ is based on two mappings \hat{S}_2 and \mathcal{D}_2 defined as: $\hat{S}_2(e_i) \leq \min [\hat{S}_1(\tilde{v}_i), \hat{S}_1(\tilde{v}_j)]$ and $\mathcal{D}_2(e_i) \leq \max [\mathcal{D}_1(\tilde{v}_i), \mathcal{D}_1(\tilde{v}_j)]$ with a restriction that $0 \leq \hat{S}_2 + \mathcal{D}_2 \leq 1$.

2.2.1 Example

The graph in Fig. 1 is an example of IFG.

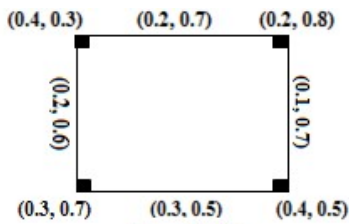


Fig. 1: IFG.

2.4 Definition [25]

A HFS is of the form $M = [(x, h(x)) | \forall x \in X]$ where $h(x)$ a set of different values in is $[0,1]$ denoted the membership values. Here $h(x)$ is a hesitant fuzzy number (HFN).

2.5 Definition [49]

A HFG is a duplet $G = (\tilde{V}, \tilde{E})$ where \tilde{V} is a set of nodes s.t and \hat{S}_1 is a map on $[0,1]$ and having the form of HFN for $\tilde{v}_n \in \tilde{V}$. Further, \tilde{E} is the set of edges where $e_i \in \tilde{V} \times \tilde{V}$ is based on a HFN \hat{S}_2 s.t. $\hat{S}_2(e_i) \leq \min [\hat{S}_1(\tilde{v}_i), \hat{S}_1(\tilde{v}_j)]$.

2.5.1 Example

The Fig. 2 is an example of HFG.

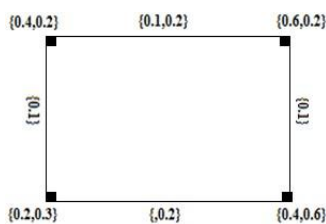


Fig. 2: HFG.

2.6 Definition [20]

A SVNNS consist some triplet of the form $\hat{A} = (\hat{S}, \tilde{I}, \mathcal{D})$ in set \tilde{X} where \hat{S}, \tilde{I} and \mathcal{D} are functions from the set \tilde{X} to an element from unit interval $[0,1]$ with a restriction that $0 \leq \hat{S} + \tilde{I} + \mathcal{D} \leq 3$. Further $\hat{A} = (\hat{S}, \tilde{I}, \mathcal{D})$ is called a single valued neutrosophic number (SVNN).

2.7 Definition [20]

Consider $\hat{A}_1 = (\hat{S}_1, \tilde{I}_1, \mathcal{D}_1)$ and $\hat{A}_2 = (\hat{S}_2, \tilde{I}_2, \mathcal{D}_2)$ be two SVNNS. Then

1. $\hat{A}_1 \oplus \hat{A}_2 = (\hat{S}_1 + \hat{S}_2 - \hat{S}_1\hat{S}_2, \tilde{I}_1\tilde{I}_2, \mathcal{D}_1\mathcal{D}_2)$.
2. $\hat{A}_1 \otimes \hat{A}_2 = (\hat{S}_1\hat{S}_2, \tilde{I}_1 + \tilde{I}_2 - \tilde{I}_1\tilde{I}_2, \mathcal{D}_1 + \mathcal{D}_2 - \mathcal{D}_1\mathcal{D}_2)$.
3. $\lambda\hat{A} = (1 - (1 - \hat{S}_1)^\lambda, \tilde{I}_1^\lambda, \mathcal{D}_1^\lambda)$.
4. $\hat{A}_1^\lambda = (\hat{S}_1^\lambda, (1 - \tilde{I}_1)^\lambda, (1 - \mathcal{D}_1)^\lambda)$ where $\lambda > 0$.

2.8 Definition [20]

The score, accuracy and certainty values of a SVNN $\hat{A}_1 = (\hat{S}_1, \tilde{I}_1, \mathcal{D}_1)$ is of the form:

1. $s(\hat{A}_1) = \frac{2+\hat{S}_1-\tilde{I}_1-\mathcal{D}_1}{3}$
2. $a(\hat{A}_1) = \hat{S}_1 - \mathcal{D}_1$
3. $c(\hat{A}_1) = \hat{S}_1$

2.9 Definition [20]

Consider $\hat{A}_1 = (\hat{S}_1, \tilde{I}_1, \mathcal{D}_1)$ and $\hat{A}_2 = (\hat{S}_2, \tilde{I}_2, \mathcal{D}_2)$ be the two SVNNS. Then:

1. $\hat{A}_1 < \hat{A}_2$ if $s(\hat{A}_1) < s(\hat{A}_2)$
2. $\hat{A}_1 > \hat{A}_2$ if $s(\hat{A}_1) > s(\hat{A}_2)$
3. $\hat{A}_1 = \hat{A}_2$ if $s(\hat{A}_1) = s(\hat{A}_2)$.

2.10 Definition [44]

An SVNG is a duplet $G = (\tilde{V}, \tilde{E})$ where \tilde{V} is a set of nodes s.t and \hat{S}_1, \tilde{I}_1 and \mathcal{D}_1 are three mappings on $[0,1]$ interval for $\tilde{v}_n \in \tilde{V}$ respectively with a restriction $0 \leq \hat{S}_1 + \tilde{I}_1 + \mathcal{D}_1 \leq 3$ and \tilde{E} is the set of edges where $e_i \in \tilde{V} \times \tilde{V}$ is based on three mappings \hat{S}_2, \tilde{I}_2 and \mathcal{D}_2 defined as:

$$\hat{S}_2(e_i) \leq \min [\hat{S}_1(\tilde{v}_i), \hat{S}_1(\tilde{v}_j)],$$

$$\tilde{I}_2(e_i) \leq \max [\tilde{I}_1(\tilde{v}_i), \tilde{I}_1(\tilde{v}_j)], \mathcal{D}_2(e_i) \leq$$

$$\max [\mathcal{D}_1(\tilde{v}_i), \mathcal{D}_1(\tilde{v}_j)] \text{ with a restriction that } 0 \leq \hat{S}_2 + \tilde{I}_2 + \mathcal{D}_2 \leq 3.$$

2.11 Definition [51]

An SVNHFS consist some triplet of the form $\tilde{N} = (\hat{S}, \tilde{I}, \mathcal{D})$ in set \tilde{X} where \hat{S}, \tilde{I} and \mathcal{D} are sets of values from the set \tilde{X} to an element from unit interval $[0,1]$ with the restrictions that $0 \leq \alpha, \beta, \gamma \leq 1$ and $0 \leq \alpha^+, \beta^+, \gamma^+ \leq 3$ where $\alpha \in \hat{S}, \beta \in \tilde{I}, \gamma \in \mathcal{D}, \alpha^+ \in \hat{S}^+ = \max \cup_{\alpha \in \hat{S}}, \beta^+ \in \tilde{I}^+ = \max \cup_{\beta \in \tilde{I}},$ and $\gamma^+ \in \mathcal{D}^+ = \max \cup_{\gamma \in \mathcal{D}}$. Further $\tilde{N} = (\hat{S}, \tilde{I}, \mathcal{D})$ is called a single valued neutrosophic hesitant fuzzy number (SVNHFN).

2.11.1 Example

The Fig. 3 is an example of SVNG.



Fig. 3: SVNG.

3. Single Valued Neutrosophic Hesitant Fuzzy Graphs

In this section we will discuss the new concept of SVNHFG as a generalization of SVNG and HFG along with some important results and their properties.

3.1 Definition

A SVNHFG is a pair $G = (\tilde{V}, \tilde{E})$ where \tilde{V} is a set of nodes s.t and \hat{S}_1, \tilde{I}_1 and \mathcal{D}_1 are three HFNs that described each $\tilde{v}_n \in \tilde{V}$ respectively with a restriction that $0 \leq \max \hat{S}_1 + \max \tilde{I}_1 + \max \mathcal{D}_1 \leq 3$ and \tilde{E} is the set of edges where $e_i \in \tilde{V} \times \tilde{V}$ is based on three HFNs \hat{S}_2, \tilde{I}_2 and \mathcal{D}_2 defined as:

$$\hat{S}_2(e_i) \leq \min [\hat{S}_1(\tilde{v}_i), \hat{S}_1(\tilde{v}_j)],$$

$$\tilde{I}_2(e_i) \leq \max [\tilde{I}_1(\tilde{v}_i), \tilde{I}_1(\tilde{v}_j)],$$

$$\mathcal{D}_2(e_i) \leq \max [\mathcal{D}_1(\tilde{v}_i), \mathcal{D}_1(\tilde{v}_j)] \text{ with a restriction that } 0 \leq \max \hat{S}_2 + \max \tilde{I}_2 + \max \mathcal{D}_2 \leq 3.$$

3.1.1 Example

The Fig. 4 is an example of SVNHFG.

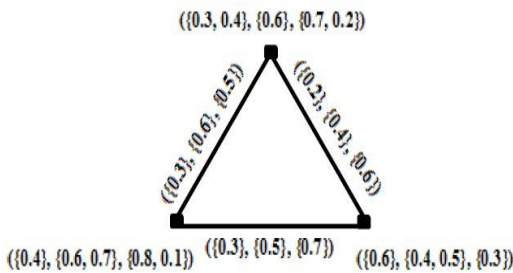


Fig. 4: SVNHFG.

3.2 Definition

A pair $S = (V', \tilde{E}')$ is a SVNHF subgraph of a SVNHFG $G = (\tilde{V}, \tilde{E})$ if

1. $\hat{S}'_{1i} \leq \hat{S}_{1i}, \tilde{I}'_{1i} \leq \tilde{I}_{1i}, \mathcal{D}'_{1i} \leq \mathcal{D}_{1i}$.
 2. $\hat{S}'_{2ij} \leq \hat{S}_{2ij}, \tilde{I}'_{2ij} \leq \tilde{I}_{2ij}, \mathcal{D}'_{2ij} \leq \mathcal{D}_{2ij}$.
- $\forall i, j = 1, 2, 3, \dots, n$.

3.3 Definition

A SVNHFG is said to be complete SVNHFG if

$$\hat{S}_{2ij} = \min(\hat{S}_{1i}, \hat{S}_{1j})$$

$$\tilde{I}_{2ij} = \max(\tilde{I}_{1i}, \tilde{I}_{1j})$$

$$\mathcal{D}_{2ij} = \max(\mathcal{D}_{1i}, \mathcal{D}_{1j}) \forall e_i \in \tilde{E}.$$

3.3.1 Example

The Fig. 5 is an example of complete SVNHF.

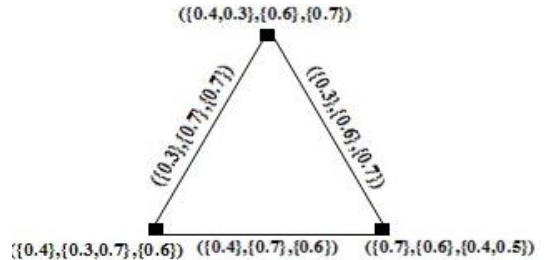


Fig. 5: Complete SVNHFG.

3.4 Definition

The complement of SVNHFG $G = (\tilde{V}, \tilde{E})$ is a SVNHFG $\bar{G} = (\bar{\tilde{V}}, \bar{\tilde{E}})$ where

1. $\bar{\tilde{V}} = \tilde{V}$ i.e. $\bar{\hat{S}}_1(\tilde{v}_i) = \hat{S}_1(\tilde{v}_i), \bar{\tilde{I}}_1(\tilde{v}_i) = \tilde{I}_1(\tilde{v}_i)$ and $\bar{\mathcal{D}}_1(\tilde{v}_i) = \mathcal{D}_1(\tilde{v}_i)$

$$\bar{\hat{S}}_2(e_i) = \min[\hat{S}_1(\tilde{v}_i), \hat{S}_1(\tilde{v}_j)] - \hat{S}_2(e_i)$$

$$\bar{\tilde{I}}_2(e_i) = \max[\tilde{I}_1(\tilde{v}_i), \tilde{I}_1(\tilde{v}_j)] - \tilde{I}_2(e_i)$$

$$\bar{\mathcal{D}}_2(e_i) = \max[\mathcal{D}_1(\tilde{v}_i), \mathcal{D}_1(\tilde{v}_j)] - \mathcal{D}_2(e_i) \forall \tilde{v}_i, \tilde{v}_j \in \tilde{V} \text{ and } e_i \in \tilde{E}.$$

3.4.1 Proposition

The complement of every SVNHFG is a SVNHFG.

3.4.2 Example

Fig. 6 shows SVNHFG, while Fig. 7 represents its complement.

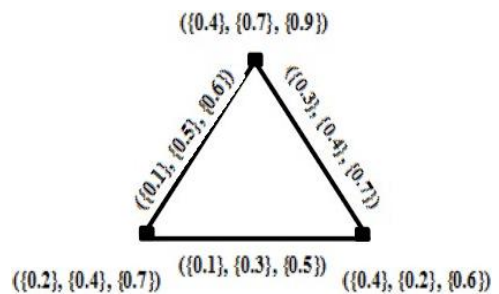


Fig. 6: SVNHFG.

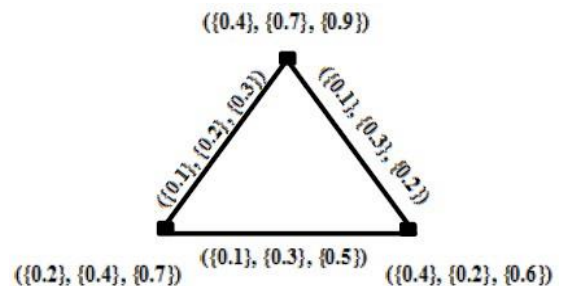


Fig. 7: Complement of SVNHFG.

3.5 Definition

The degree of vertex of a SVNHFG is defined by $\check{d}(\check{v}) = (\check{d}_S(\check{v}), \check{d}_I(\check{v}), \check{d}_D(\check{v}))$ where,

$$\check{d}_S(\check{v}) = \sum_{\check{v} \neq w} \hat{S}_2(\check{v}, w)$$

$$\check{d}_I(\check{v}) = \sum_{\check{v} \neq w} \hat{I}_2(\check{v}, w)$$

$$\check{d}_D(\check{v}) = \sum_{\check{v} \neq w} \hat{D}_2(\check{v}, w)$$

where \check{d}_S, \check{d}_I and \check{d}_D denote the degree \hat{S}, \hat{I} and \hat{D} , respectively.

3.6 Definition

The maximum degree of SVNHFG is defined by $\nabla(G) = (\nabla_S(G), \nabla_I(G), \nabla_D(G))$ where, $\nabla_S(G) = \vee \{\check{d}_S(\check{v}); \check{v} \in \check{V}\}$

$$\nabla_I(G) = \vee \{\check{d}_I(\check{v}); \check{v} \in \check{V}\} \text{ and}$$

$$\nabla_D(G) = \vee \{\check{d}_D(\check{v}); \check{v} \in \check{V}\} \text{ where } \nabla_S, \nabla_I \text{ and } \nabla_D \text{ denotes the degree of maximum } \hat{S}, \hat{I} \text{ and } \hat{D}, \text{ respectively.}$$

The minimum degree of SVNHFG is defined by

$$\partial(G) = (\partial_S(G), \partial_I(G), \partial_D(G)) \text{ where}$$

$$\partial_S(G) = \wedge \{\check{d}_S(\check{v}); \check{v} \in \check{V}\}$$

$$\partial_I(G) = \wedge \{\check{d}_I(\check{v}); \check{v} \in \check{V}\}$$

$$\partial_D(G) = \wedge \{\check{d}_D(\check{v}); \check{v} \in \check{V}\} \text{ where } \partial_S, \partial_I \text{ and } \partial_D \text{ denote the degree of minimum } \hat{S}, \hat{I} \text{ and } \hat{D}, \text{ respectively.}$$

3.6.1 Example

The Fig. 8 is a SVNHFG where degree of vertices is computed using definition (3.9).

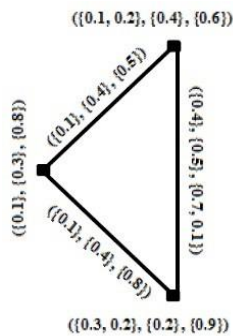


Fig. 8: Degree of SVNHFG.

$$\check{d}_S(\check{v}_1) = 0.2, \check{d}_I(\check{v}_1) = 0.7, \check{d}_D(\check{v}_1) = 1.3.$$

$$d(\check{v}_1) = (\{0.2\}, \{0.7\}, \{1.3\})$$

$$d(\check{v}_2) = (\{0.2\}, \{0.7\}, \{1.6\})$$

$$d(\check{v}_3) = (\{0.2\}, \{0.8\}, \{1.3\})$$

$$\partial(G) = (\{0.2\}, \{0.7\}, \{1.3\})$$

$$\nabla(G) = (\{0.2\}, \{0.8\}, \{1.6\})$$

3.6.2 Proposition

In a SVNHFG $G = (\check{V}, \check{E})$, the $\sum \check{d}(\check{v}_i) = [2 \sum_{\check{v} \neq w} \check{d}(\check{v}, w)]$.

Proof:

Let $G = (\check{V}, \check{E})$ be a SVNHFG.

Then

$$\begin{aligned} \sum \check{d}(\check{v}_i) &= \sum [\check{d}_S(\check{v}_i), \check{d}_I(\check{v}_i), \check{d}_D(\check{v}_i)] \\ &= [\sum \check{d}_S(\check{v}_i), \sum \check{d}_I(\check{v}_i), \sum \check{d}_D(\check{v}_i)] \text{ where } i = 1, 2, 3 \dots n. \\ &= [\sum_{i \neq j} \hat{S}_2(e_i), \sum_{i \neq j} \hat{I}_2(e_i), \sum_{i \neq j} \hat{D}_2(e_i)] \\ &= [\sum_{i \neq j} \hat{S}_2(e_i), \sum_{i \neq j} \hat{I}_2(e_i), \sum_{i \neq j} \hat{D}_2(e_i)] \\ &= [(\hat{S}_2(\check{v}_1, \check{v}_2), \hat{I}_2(\check{v}_1, \check{v}_2), \hat{D}_2(\check{v}_1, \check{v}_2)) + (\hat{S}_2(\check{v}_1, \check{v}_3), \hat{I}_2(\check{v}_1, \check{v}_3), \hat{D}_2(\check{v}_1, \check{v}_3)) + \dots \\ &\quad + (\hat{S}_2(\check{v}_1, \check{v}_n), \hat{I}_2(\check{v}_1, \check{v}_n), \hat{D}_2(\check{v}_1, \check{v}_n)) \\ &\quad + (\hat{S}_2(\check{v}_2, \check{v}_1), \hat{I}_2(\check{v}_2, \check{v}_1), \hat{D}_2(\check{v}_2, \check{v}_1)) \\ &\quad + (\hat{S}_2(\check{v}_2, \check{v}_3), \hat{I}_2(\check{v}_2, \check{v}_3), \hat{D}_2(\check{v}_2, \check{v}_3)) + \dots \\ &\quad + (\hat{S}_2(\check{v}_2, \check{v}_n), \hat{I}_2(\check{v}_2, \check{v}_n), \hat{D}_2(\check{v}_2, \check{v}_n)) \\ &\quad + \dots (\hat{S}_2(\check{v}_n, \check{v}_1), \hat{I}_2(\check{v}_n, \check{v}_1), \hat{D}_2(\check{v}_n, \check{v}_1)) \\ &\quad + \dots (\hat{S}_2(\check{v}_n, \check{v}_{n-1}), \hat{I}_2(\check{v}_n, \check{v}_{n-1}), \hat{D}_2(\check{v}_n, \check{v}_{n-1})))] \\ &= [2 \sum_{i \neq j} \hat{S}_2(e_i), 2 \sum_{i \neq j} \hat{I}_2(e_i), 2 \sum_{i \neq j} \hat{D}_2(e_i)] \end{aligned}$$

Hence proof.

3.6.3 Proposition

The maximum degree of any vertex in a SVNHFG with $n + 1$ vertex is n .

Proof:

Consider a SVNHFG. As $\max (\hat{S}_2(\check{v}_k, \check{v}_1)) = 1$ and maximum number of edges are 'n' of 'n + 1' vertices. Then $\check{d}_S(\check{v}_k) = n$ (In SVNHFG).

Similarly,

$$\check{d}_I(\check{v}_n) = n$$

$$\check{d}_D(\check{v}_n) = n,$$

Proved.

3.7 Definition

In a SVNHFG $G = (\check{V}, \check{E})$ the path P is sequence of distinct vertices $\check{v}_1, \check{v}_2, \dots \check{v}_n$ such that either

$$\hat{S}_2(\check{v}_{i-1}, \check{v}_i) > 0$$

or

$$\hat{I}_2(\check{v}_{i-1}, \check{v}_i) > 0$$

or

$$\hat{D}_2(\check{v}_{i-1}, \check{v}_i) > 0$$

for $0 \leq i \leq 1$.

3.8 Definition

In a SVNHFG $G = (\check{V}, \check{E})$ the path P is sequence of distinct vertices $\check{v}_1, \check{v}_2, \dots \check{v}_n$ Where $n \geq 1$ is called the length of the

path. A single vertex \tilde{v}_i is also a path but in this case the length of the path is $(\{0\}, \{0\}, \{0\})$.

3.9 Definition

A SVNHFG $G = (\tilde{V}, \tilde{E})$ is connected if every two vertices have a SVNHF path between them otherwise disconnected.

3.10 Definition

In a SVNHFG $G = (\tilde{V}, \tilde{E})$, the consecutive pairs $(\tilde{v}_{i-1}, \tilde{v}_i)$ are called edges of path.

3.11 Definition

In a SVNHFG $G = (\tilde{V}, \tilde{E})$, the path P is called a cycle if $\tilde{v}_0 = \tilde{v}_n$ where $n \geq 3$.

3.12 Definition

In a SVNHFG $G = (\tilde{V}, \tilde{E})$ if $\hat{S}_2 = \tilde{I}_2 = \mathcal{D}_2 = 0$ then there does not exist any edge.

4. Algorithm for finding shortest path in SVNHFG

To compute the shortest path from each node to its predecessor, an algorithm is proposed in this section. In real life problem this algorithm is useful to find the shortest path in a network.

Step 1:

Identify the first and final nodes of destination as \tilde{v}_1 and \tilde{v}_n .

Step 2:

Take $\tilde{d}_1 = (\{0\}, \{1\}, \{1\})$ as there is no distance of node 1 from itself. Further, label the first node as $(\{0\}, \{1\}, \{1\}, -)$.

Step 3:

Find $\tilde{d}_j = \min\{\tilde{d}_i \oplus \tilde{d}_{ij}\}$. For $j = 2, 3, \dots, n$. Since the numbers are SVNHFNs so here we use mean value of function instead of using each value of function. That is $\hat{S}_1 = \frac{\hat{S}_{11} + \hat{S}_{12} + \dots + \hat{S}_{1n}}{n}$, where $n = 1, 2, 3 \dots n$.

Step 4:

If the value of distance occurs against a unique $i = r$. Then j is marked as $[\tilde{d}_j, r]$.

If the values of distance do not occur against a unique i . It represents more than one SVNHF paths from a node. So, to get the shortest among several paths, use the score function of SVNHFNs.

Step 5:

Let $[\tilde{d}_n, k]$ is the label of the destination node then the shortest displacement between initial and final node is \tilde{d}_n .

Step 6:

Since $[\tilde{d}_n, k]$ is the label as destination node. So, for finding SVNHF shortest path from first node to last node, we check the label of node k . Let it be $[\tilde{d}_n, l]$. Then we check the label of node l and so on. Repeat this process to obtain the initial node.

Step 7:

Hence, the SVNHF shortest path can be obtained by using step 6.

4.1 Example

Consider a network based on SVNHG given in Fig. 9 where the distance between the vertices is a SVNHFN. Using the proposed approach, the shortest path is computed as follows:

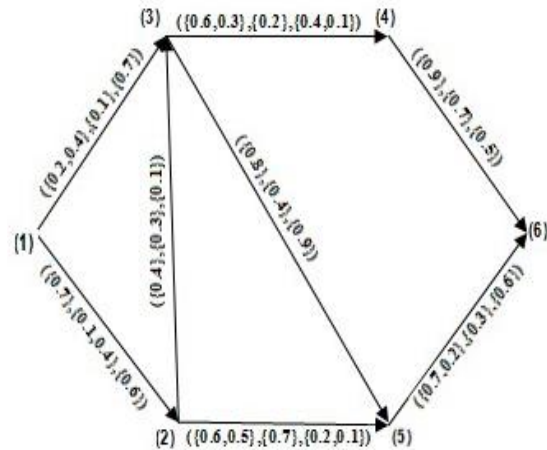


Fig. 9: SVNHF network.

The path between every two nodes is described in Table 1 using SVNHFNs.

Table 1: (SVNHF Network edges).

Edges	Distances
1-2	$(\{0.7\}, \{0.4, 0.1\}, \{0.6\})$
1-3	$(\{0.2, 0.4\}, \{0.1\}, \{0.7\})$
2-3	$(\{0.4\}, \{0.3\}, \{0.1\})$
2-5	$(\{0.6, 0.5\}, \{0.7\}, \{0.2, 0.1\})$
3-4	$(\{0.3, 0.6\}, \{0.2\}, \{0.4, 0.1\})$
3-5	$(\{0.8\}, \{0.4\}, \{0.9\})$
4-6	$(\{0.9\}, \{0.7\}, \{0.5\})$
5-6	$(\{0.7, 0.2\}, \{0.3\}, \{0.6\})$

Now we compute the shortest path by using the described algorithm as follows:

As the destination node is 6 so $n = 6$. Let $\tilde{d}_1 = (\{0\}, \{1\}, \{1\})$ and mark the source node as $(\{0\}, \{1\}, \{1\}, -)$ (say node 1), \tilde{d}_j can be find as follows.

Iteration 1:

Since the only predecessor of node 2 is 1, put $i = 1$ and $\hat{j} = 2$, by algorithm we get \tilde{d}_2 as

$$\begin{aligned} \tilde{d}_2 &= \min\{\tilde{d}_1 \oplus \tilde{d}_{12}\} \\ &= \min((\{0\}, \{1\}, \{1\}) \oplus (\{0.7\}, \{0.4, 0.1\}, \{0.6\})) \\ &= \min((\{0\}, \{1\}, \{1\}) \oplus ((0.7), (0.25), (0.6))) \\ &= (\{0.7\}, \{0.25\}, \{0.6\}) \end{aligned}$$

Minimum occurs for one value of $i = 1$. So, vertex 2 is labeled as $[(\{0.7\}, \{0.25\}, \{0.6\}), -1]$

Iteration 2:

Since 1 and 2 are the predecessor of node 3 so

Put $i = 1, 2$ and $j = 3$. By algorithm

$$\begin{aligned} \bar{d}_3 &= \min\{\bar{d}_1 \oplus \bar{d}_{13}, \bar{d}_2 \oplus \bar{d}_{23}\}. \\ &= \min\{(\{0\}, \{1\}, \{1\}) \\ &\quad \oplus (\{0.2, 0.4\}, \{0.1\}, \{0.7\}), (\{0.7\}, \{0.25\}, \{0.6\}) \\ &\quad \oplus (\{0.4\}, \{0.3\}, \{0.1\})\}. \\ &= \min\{(\{0.3\}, \{0.1\}, \{0.7\}), (\{0.82\}, \{0.08\}, \{0.06\})\}. \end{aligned}$$

By using score function, we can get the minimum:

$$s(\{0.3\}, \{0.1\}, \{0.7\}) = 0.5 \text{ and}$$

$$s(\{0.82\}, \{0.08\}, \{0.06\}) = 0.89.$$

So, the $\bar{d}_3 = (\{0.3\}, \{0.1\}, \{0.7\})$.

The minimum occurs for $i = 1$. So, the vertex 3 is labeled as $[(\{0.3\}, \{0.1\}, \{0.7\}), 1]$.

Iteration 3:

Node 3 is the predecessor of node 4 so put $i = 3$ and $j = 4$, by algorithm

$$\begin{aligned} \bar{d}_4 &= \min\{\bar{d}_3 \oplus \bar{d}_{34}\} \\ &= \min\{(\{0.3\}, \{0.1\}, \{0.7\}) \oplus \\ &\quad (\{0.3, 0.6\}, \{0.2\}, \{0.4, 0.1\})\} \\ &= \min\{(\{0.3\}, \{0.1\}, \{0.7\}) \oplus \\ &\quad (\{0.45\}, \{0.2\}, \{0.25\})\} = (\{0.61\}, \{0.02\}, \{0.18\}) \end{aligned}$$

Minimum occurs for $i = 3$. So, the vertex 4 is labeled as $[(\{0.61\}, \{0.02\}, \{0.18\}), 3]$.

Iteration 4:

Node 2 and node 3 are predecessor of node 5 so put $i = 2, 3$ and $j = 5$. By algorithm

$$\begin{aligned} \bar{d}_5 &= \min\{\bar{d}_2 \oplus \bar{d}_{25}, \bar{d}_3 \oplus \bar{d}_{35}\} \\ &= \min\{(\{0.7\}, \{0.25\}, \{0.6\}) \oplus (\{0.6, 0.5\}, \{0.7\}, \{0.2, 0.1\}), \\ &\quad (\{0.3\}, \{0.1\}, \{0.7\}) \oplus (\{0.8\}, \{0.4\}, \{0.9\})\} \\ &= \min\{(\{0.7\}, \{0.25\}, \{0.6\}) \oplus (\{0.55\}, \{0.7\}, \{0.15\}), \\ &\quad (\{0.3\}, \{0.1\}, \{0.7\}) \oplus (\{0.8\}, \{0.4\}, \{0.9\})\} \\ &= \min\left\{\begin{aligned} &(\{0.88\}, \{0.18\}, \{0.09\}), \\ &(\{0.86\}, \{0.04\}, \{0.63\}) \end{aligned}\right\} \end{aligned}$$

By score function we can get that,

$$\bar{d}_5 = (\{0.86\}, \{0.04\}, \{0.63\})$$

Minimum occurs for $i = 3$. So, vertex 5 is labeled as $[(\{0.86\}, \{0.04\}, \{0.63\}), 3]$.

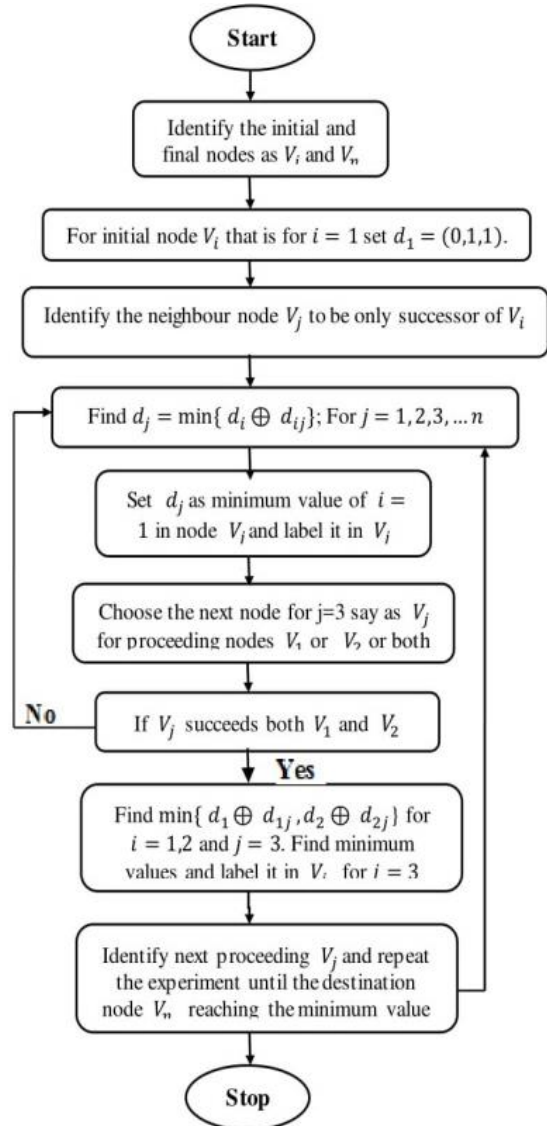


Fig. 10: (Flowchart of algorithm).

Iteration 5:

Node 4 and 5 are the predecessor of node 6 so put $i = 4, 5$ and $j = 5$ by algorithm,

$$\begin{aligned} \bar{d}_6 &= \min\{\bar{d}_4 \oplus \bar{d}_{46}, \bar{d}_5 \oplus \bar{d}_{56}\}. \\ &= \min\{(\{0.61\}, \{0.02\}, \{0.18\}) \oplus (\{0.9\}, \{0.7\}, \{0.5\}), \\ &\quad (\{0.86\}, \{0.04\}, \{0.63\}) \oplus (\{0.7, 0.2\}, \{0.3\}, \{0.6\})\}. \\ &= \min\{(\{0.61\}, \{0.02\}, \{0.18\}) \oplus (\{0.9\}, \{0.7\}, \{0.5\}), \\ &\quad (\{0.86\}, \{0.04\}, \{0.63\}) \oplus (\{0.45\}, \{0.3\}, \{0.6\})\}. \\ &= \min\{(\{0.96\}, \{0.01\}, \{0.09\}), \\ &\quad (\{0.92\}, \{0.01\}, \{0.38\})\} \end{aligned}$$

By score function we get that

$$\bar{d}_6 = (\{0.92\}, \{0.01\}, \{0.38\})$$

Minimum occurs for $i = 5$. So, vertex 6 is labeled as $[(\{0.92\}, \{0.01\}, \{0.38\}), 5]$.

Since the destination point is \bar{d}_6 . So, the shortest displacement from vertex one to six is provided as:

$$(\{0.92\}, \{0.01\}, \{0.38\}).$$

The shortest way can be determined as follows:

Node 6 is labelled as $[(\{0.92\}, \{0.01\}, \{0.38\}), 5]$

Node 5 is labelled as $[(\{0.86\}, \{0.04\}, \{0.63\}), 3]$

Node 3 is labelled as $[(\{0.61\}, \{0.02\}, \{0.18\}), 1]$.

Hence, the shortest way is $1 \rightarrow 3 \rightarrow 5 \rightarrow 6$

with the SVNHF value of distances is

$$(\{0.92\}, \{0.01\}, \{0.38\}).$$

In Fig. 10 the dotted line represents the shortest path and Table 2 provides the path of different nodes.

Table 2: Shortest path.

Nodes No. (j)	d_i	Shortest path from 1 st node to j th node
2	$(\{0.7\}, \{0.25\}, \{0.6\})$	$1 \rightarrow 2$
3	$(\{0.3\}, \{0.1\}, \{0.7\})$	$1 \rightarrow 3$
4	$(\{0.61\}, \{0.02\}, \{0.18\})$	$1 \rightarrow 3 \rightarrow 4$
5	$(\{0.86\}, \{0.04\}, \{0.63\})$	$1 \rightarrow 3 \rightarrow 5$
6	$(\{0.92\}, \{0.01\}, \{0.38\})$	$1 \rightarrow 3 \rightarrow 5 \rightarrow 6$

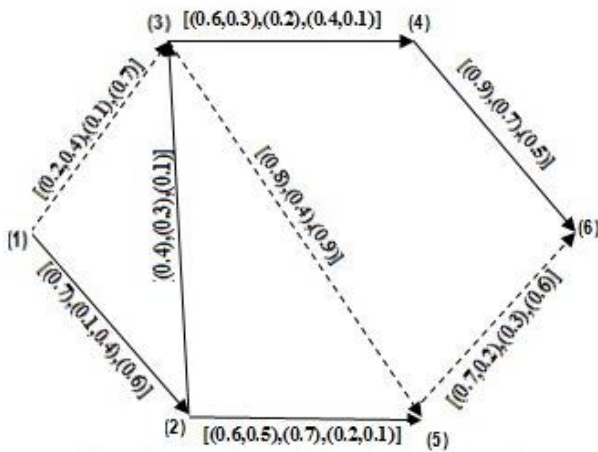


Fig. 11: Shortest path network of SVNHFNs.

5. Conclusions

In this article, an algorithm for computing the shortest path in a network having nodes in the form of SVNHFNs is developed. The key findings of the manuscript are as under:

- The concept of SVNHFG is introduced which is an extension of neutrosophic graph and hesitant fuzzy graph.
- The terms like complement, subgraphs, degree and path in SVNHFGs are defined and supported with examples.
- An algorithm for computing shortest path is proposed which is further demonstrated by a flowchart.

- A numerical example is solved using the proposed algorithm where the shortest path in a network is computed among all possible paths.
- It is observed that the results obtained in the manuscript are compatible with that of presented in Broumi et al. [9, 10, 12, 13, 14].

In future, our aim is to extend our work to produce some more interesting graph theoretic results in the environment of SVNHFGs and try to utilize them in multi-attribute decision making and supply chain management problems.

References

- R. Dial, F. Glover, D. Karney and D. Klingman, "A computational analysis of alternative algorithms and labeling techniques for finding shortest path trees", *Networks*, vol. 9, no. 3, pp. 215-248, 1979.
- G.Y. Handler and I. Zang, "A dual algorithm for the constrained shortest path problem", *Networks*, vol. 10, no. 4, pp. 293-309, 1980.
- Z. Teradata, "Selected multicriteria shortest path problems: An analysis of complexity, models and adaptation of standard algorithms", *Int. J. Ap. Mat. Com-pol.*, vol. 17, no. 2, pp. 269-287, 2007.
- S. Okada and S. Soper, "A shortest path problem on a network with fuzzy arc lengths", *Fuzzy Set Syst.*, vol. 109, no. 1, pp. 129-140, 2000.
- K.C. Lin and M.S. Chern, "The fuzzy shortest path problem and its most vital arcs", *Fuzzy Set Syst.*, vol. 58, no. 3, pp. 343-353, 1993.
- Y. Deng, Y. Chen, Y. Zhang and S. Mahadevan, "Fuzzy Dijkstra algorithm for shortest path problem under uncertain environment", *Appl. Soft. Comput.*, vol. 12, no. 3, pp. 1231-1237, 2012.
- S.N. Chuang and J.Y. Kung, "The fuzzy shortest path length and the corresponding shortest path in a network", *Comp. Oper. Res.*, vol. 32, no. 6, pp. 1409-1428, 2005.
- S. Broumi, A. Bakal, M. Talea, F. Smarandache and L. Vladareanu. "Applying Dijkstra algorithm for solving neutrosophic shortest path problem", *Int. Conf. Adv. Mechatronic Syst.*, vol. 2, pp. 412-416, November 2016.
- S. Broumi, A. Bakal, M. Talea, F. Smarandache and L. Vladareanu, "Computation of shortest path problem in a network with SV-trapezoidal neutrosophic numbers", *Int. Conf. Adv. Mechatronic Syst.*, vol. 2, pp. 417-422, November 2016.
- S. Broumi, A. Bakal, M. Talea, F. Smarandache and M. Ali, "Shortest Path Problem under Bipolar Neutrosophic Setting", *Appl. Mech. Mater.*, vol. 859, pp. 59-66, 2017.
- S. Broumi, A. Bakal, M. Talea, F. Smarandache and K.K. PK, "Shortest path problem on single valued neutrosophic graphs", *Infinite Study*, vol. 2, no. 1, pp. 1-6, 2017.
- S. Broumi, A. Bakal, M. Talea, F. Smarandache, K.K. Kishore and R. Sahin, "Shortest path problem under interval valued neutrosophic setting", *J. Fundam. Appl. Sci.*, vol. 10, no. 4, pp. 168-174, 2018.
- S. Broumi, A. Bakal, M. Talea, F. Smarandache and K. Ullah, "Bipolar Neutrosophic Minimum Spanning Tree", *Infinite Study*, vol. 1, pp. 2, 2018.
- J. Ye, "Single-valued neutrosophic minimum spanning tree and its clustering method", *J. Intell. Syst.*, vol. 23, no. 3, pp. 311-324, 2014.
- L.A. Zadeh, "Fuzzy sets", *Inf. Control.*, vol. 8, no. 3, pp. 338-353, 1965.
- K.S. Atanassov, "Intuitionistic fuzzy sets." *Fuzzy sets and Syst.*, vol. 20, no. 1, pp. 87-96, 1986.
- Mukherjee, Anjan and S. Sarkar, "Several similarity measures of interval valued neutrosophic soft sets and their application in pattern recognition problems", *Neutrosophic Sets Syst.*, vol. 6, pp. 55-61, 2014.
- W. Haibin, F. Smarandache, Y. Zhang, and R. Sunderaman, "Single valued neutrosophic sets", *Infinite Study*, vol. 17, no. 1, pp. 10, 2010.
- B.C. Cuong, "Picture fuzzy sets", *J. Comp. Sci. Cyberne.*, vol. 30, no. 4, pp. 409, 2014.
- T. Mahmood, K. Ullah, Q. Khan and N. Jan, "An approach towards decision making and medical diagnosis problems using the concept of spherical fuzzy sets", *Neural Comput. Appl.*, vol. 31, no. 11, pp. 7041-7053, 2018.

- [21] K. Ullah, T. Mahmood and N. Jan, "Similarity Measures for T-Spherical fuzzy sets with applications in pattern recognition", *Symmetry*, vol. 10, no. 6, pp. 193, 2018.
- [22] V. Torra, "Hesitant fuzzy sets", *Int. J. Intell. Syst.*, vol. 25, no. 6, pp. 529-539, 2010.
- [23] K. Ullah, Mahmood, N. Jan, S. Broumi S and Q. Khan, "On Bipolar-valued hesitant fuzzy sets and their applications in multi-attribute decision making", *The Nucleus*, vol. 55, no. 2, pp. 93-101, 2018.
- [24] T. Mahmood, K. Ullah, Q. Khan and F. Smarandache, "Some aggregation operators for bipolar-valued hesitant fuzzy information", *J Fundam. Appl. Sci.*, vol. 10, no. 4, pp. 240-245, 2018.
- [25] T. Mahmood, J. Ye, and Q. Khan, "Vector similarity measures for simplified neutrosophic hesitant fuzzy set and their applications", *J Inequ. Spec. Func.*, vol. 7, no. 4, pp. 176-194, 2016.
- [26] T. Mahmood and M. Munir, "On bipolar fuzzy subgroups", *World Appl. Sci. J*, vol. 27, no. 12, pp. 1806-1811, 2013.
- [27] Q. Khan, T. Mahmood and J. Ye, "Multiple attribute decision-making method under hesitant single valued neutrosophic uncertain linguistic environment", *Infinite Study*, vol. 8, no. 2, pp. 1-17, 2017.
- [28] N. Jan, L. Zedam, T. Mahmood, K. Ullah, Z. Ali, "Multiple attribute decision making method under linguistic cubic information", *J Intell. Fuzzy Syst.*, vol. 36, no. 1, pp. 253-269, 2019.
- [29] A. Kaufmann, "Introduction à la théorie des sous-ensembles flous à l'usage des ingénieurs", *Éléments théoriques de base*, vol. 1, pp. 41-189, 1973.
- [30] A. Rosenfeld, "Fuzzy graphs, in Fuzzy sets and their applications to cognitive and decision processes", Elsevier, 1975.
- [31] R. Parvathi and M. Karunambigai, "Intuitionistic fuzzy graphs, in Computational Intelligence, Theory and Applications", Springer, 2006.
- [32] R. Parvathi, M. Karunambigai and K. Atanassov. "Operations on intuitionistic fuzzy graphs", *Fuzzy Syst. IEEE Int. Conf. Fuzzy Syst.*, vol. 51, no. 5, pp. 1396-1409, 2009.
- [33] R. Parvathi, S. Thilagavathi and M. Karunambigai, "Intuitionistic fuzzy hypergraphs", *Cyberne. Info. Tech.*, vol. 9, no. 2, pp. 46-53, 2009.
- [34] R. Parvathi, S. Thilagavathi, G. Thamizhendhi and M.G. Karunambigai, "Index matrix representation of intuitionistic fuzzy graphs", *Notes Intuitionistic Fuzzy Sets*, vol. 20, no. 2, pp. 100-108, 2014.
- [35] G. Pasi, R. Yager, and K. Atanassov. "Intuitionistic fuzzy graph interpretations of multi-person multi-criteria decision making", *2nd Int. IEEE Conf. Gen Net Appro. Intell. Syst.*, vol. 2, pp. 434-439, 2004.
- [36] S.S. Dhavudh and R. Srinivasan, "Intuitionistic fuzzy graphs of second type", *Adv. Fuzzy Math.*, vol. 12, no. 2, pp. 197-204, 2017.
- [37] B. Davvaz, N. Jan, T. Mahmood and K. Ullah, "Intuitionistic fuzzy graphs of nth type with applications", *J Intell. Fuzzy Syst.*, vol. 36, no. 4, pp. 3923-3932, 2018.
- [38] Yaqoob, Naveed, M. Gulistan, S. Kadry and H.A. Wahab, "Complex intuitionistic fuzzy graphs with application in cellular network provider companies", *J Math.*, vol. 7, no. 1, pp. 35, 2019.
- [39] Hussain, S. Satham, R.J. Hussain, Y.B Jun and F. Smarandache, "Neutrosophic bipolar vague set and its application to neutrosophic bipolar vague graphs", *Neutrosophic Sets Syst.*, vol. 28, no. 1, pp. 8, 2019.
- [40] S. Broumi, M. Talea, A. Bakali and F. Smarandache, "Single valued neutrosophic graphs", *J New theory*, vol. 10, pp. 86-101, 2015.
- [41] S. Broumi, M. Talea, A. Bakali and F. Smarandache, "Interval valued neutrosophic graphs", *Crit. Rev.*, vol. 10, pp. 5-33, 2016.
- [42] S. Broumi, M. Talea, A. Bakali and F. Smarandache, "On strong interval valued neutrosophic graphs", *Crit. Rev.*, vol. 12, pp. 49-71, 2016.
- [43] S. Broumi, K. Ullah, A. Bakali and M. Talea, "Novel system and method for telephone network planning based on neutrosophic graph", *Infinite Study*, vol. 10, no. 4, pp. 403-434, 2018.
- [44] M. Gulistan, N. Yaqoob, Z. Rashid, F. Smarandache and H. A. Wahab, "A study on neutrosophic cubic graphs with real life applications in industries", *Symmetry*, vol. 10, no. 6, pp. 203, 2018.
- [45] C. Zhang and D. Li, "Hesitant fuzzy graph and its application in multi-attribute decision making", *Int. J. Patran. Recogn.* vol. 30, no. 11, pp. 1012-1018, 2017.
- [46] Rashid, Sheikh, N. Yaqoob, M. Akram and M. Gulistan, "Cubic graphs with application", *Int. J. Anal. Applications*, vo. 16, no. 5, pp. 733-750, 2018.
- [47] J. Ye, "Multiple-attribute decision-making method under a single-valued neutrosophic hesitant fuzzy environment", *J Intell. Fuzzy. Syst.*, vol. 24, no. 1, pp. 23-36, 2015.