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Computing Shortest Path in a Single Valued Neutrosophic Hesitant Fuzzy Network

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ABSTRACT

In engineering, computer sciences and many other applied sciences, finding shortest path in a network is one of the famous problems. The aim of this manuscript is to develop a novel algorithm for finding shortest path in a network where nodes and edges have some uncertainty. Firstly, the concept of single-valued neutrosophic hesitant fuzzy graph (SVNHFG) has been introduced with some related graph theoretical results such that complement, subgraph, degree and path etc. Some examples are provided to understand the defined concepts. Then, the new algorithm for solving shortest path problems (SPPs) has been introduced followed by a flowchart for a stepwise description. A numerical example is provided in the environment of SVNHFGs to demonstrate the proposed algorithm. The advantages of proposed method over the existing techniques are also discussed.

1. Introduction

The computation of shortest path is a basic problem in networks especially in computer, communication and some other networks in engineering sciences. In such problems, the target is to determine the shortest path between first and final vertex. These problems are discussed in computer sciences, graph theory and Applied Mathematics [1-3].

The SPP has also been discussed in fuzzy environment since the concept of fuzzy set (FS) has been introduced. In real-life problems, sometimes we face uncertainty in computation due to several unnoticeable reasons and FS has proved to be the best tool so far in handling such problems. SPP has been studied not only using fuzzy graphs (FGs) but also using intuitionistic fuzzy graphs (IFGs), neutrosophic graphs (NGs) especially single valued neutrosophic graphs (SVNGs) and various other generalized structures. For some significant work on shortest path in fuzzy environments, one may refer to previously reported literature [4-15].

In 1965, the concept of FS was intimated by Zadeh [15], which proved to be a remarkable tool for handling uncertainties that exist in real life problems. This concept was further enhanced by Atanassov [16] and the concept of intuitionistic fuzzy sets (IFSs) have been proposed. The concept of IFS has its own limitations which further enhanced the concept of neutrosophic set (NS) by Mukherjee and Sarkar [17], which leads to the concept of single valued neutrosophic set (SVNS) by Haibin et al. [18]. There are some other directions too where IFSs have been generalized such as the concept of picture fuzzy set (PFS) introduced by Cuong [19], spherical and T-spherical fuzzy sets by Mahmood et al. [20] and Ullah et al. [21]. The concept of hesitant fuzzy set (HFS) developed by Torra [22] has also a unique way of handling uncertainties in real life phenomena. Combining different fuzzy algebraic structures to get a new structure is also common in FS theory and several new directions have been reported previously [23-28].

The theory of FGs was initiated by Kauffman [29] and further deeply studied by Rosenfeld [30]. FGs have been extensively studied and several new dimensions have been introduced. The idea of IFGs was developed by Parvathi and Karunambigai [31] and further investigated by Parvathi et al. [32-34] and Pasi et al. [35]. Similarly IFG was further enhanced to IFG of second type (IFGST) by Dhavudh and Srinivasan [36]. To make the domain of IFG more viable, the concept of IFGs of nth type (IFGNT) and complex intuitionistic fuzzy graphs (CIFG) are recently proposed by Davvaz et al. [37] and Yaqoob et al. [38]. Some study related to NGs and its generalizations is investigated recently [39-44]. The study of hesitant fuzzy graphs (HFGs) is developed by Zhang and Li [45] where its several operations are defined and their applications in decision making are studied. The concept of cubic graphs is investigated by Rashid et al. [46].

The idea of single valued neutrosophic hesitant fuzzy set (SVNHFS) has been proposed by Ye [47], which is a generalization of both SVNSs and HFSs. Motivated by the work on this recent development, in this article, we aimed to develop the concept of SVNHFGs and studied the famous SPP in a network which is based on SVNHF information.

In this article, we discuss the significance of SPP in various fields especially in fuzzy algebraic structures in first section. The second section contain some basic definitions. Section three is based on some concepts of SVNHFGs and its related terms. The fourth section is based on a novel algorithm for SPP and its flowchart. In the fifth section, we describe a numerical example based on the proposed algorithm and demonstrate its every step with details. Finally, the article is summarized with discussion of some future directions.

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2. Preliminaries

This section describes some important terms which are helpful to study the paper. Here IFSs, HFSs and SVNSs are discussed and their graphs are demonstrated. Throughout our article, the terms $\hat{S}_1(\hat{S}_2)$, $\tilde{I}_1(\tilde{I}_2)$ and $Ḑ_1(Ḑ_2)$ will denote the membership, indeterminacy and non-membership functions of nodes (edges) respectively.

2.1 Definition [18]

An IFS is a duplet $\acute{A} = (\hat{S}, Ḑ)$ in set $\dot{X}$ where $\hat{S}$, $Ḑ$ are functions from the set $\dot{X}$ to an element from unit interval $[0,1]$ with a restriction that $0 \leq \hat{S} + Ḑ \leq 1$.

2.2 Definition [34]

An IFG is a duplet $Ģ = (\tilde{V}, \ddot{E})$ where $\tilde{V}$ is a set of nodes s.t. and $\hat{S}_1$, $Ḑ_1$ are two mappings on $[0, 1]$ interval for $\tilde{v}_n \in \tilde{V}$ respectively with a restriction $0 \leq \hat{S}_1 + Ḑ_1 \leq 1$ and $\ddot{E}$ is the set of edges where $e_i \in \tilde{V} \times \tilde{V}$ is based on two mappings $\hat{S}_2$ and $Ḑ_2$ defined as: $\hat{S}_2(e_i) \leq \min[\hat{S}_1(\tilde{v}_i), \hat{S}_1(\tilde{v}_j)]$ and $Ḑ_2(e_i) \leq \max[Ḑ_1(\tilde{v}_i), Ḑ_1(\tilde{v}_j)]$ with a restriction that $0 \leq \hat{S}_2 + Ḑ_2 \leq 1$.

2.2.1 Example

The graph in Fig. 1 is an example of IFG.

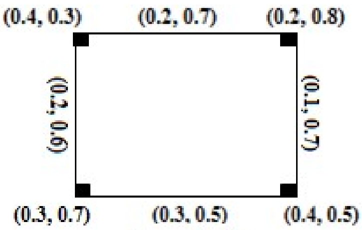

Fig. 1: IFG.

2.4 Definition [25]

A HFS is of the form $M = [(x, h(x)) | \forall x \in X]$ where $h(x)$ a set of different values in is $[0, 1]$ denoted the membership values. Here $h(x)$ is a hesitant fuzzy number (HFN).

2.5 Definition [45]

A HFG is a duplet $Ģ = (\tilde{V}, \ddot{E})$ where $\tilde{V}$ is a set of nodes s.t. and $\hat{S}_1$ is a map on $[0, 1]$ having the form of HFN for $\tilde{v}_n \in \tilde{V}$. Further, $\ddot{E}$ is the set of edges where $e_i \in \tilde{V} \times \tilde{V}$ is based on a HFN $\hat{S}_2$ s.t. $\hat{S}_2(e_i) \leq \min[\hat{S}_1(\tilde{v}_i), \hat{S}_1(\tilde{v}_j)]$.

2.5.1 Example

Fig. 2 is an example of HFG.

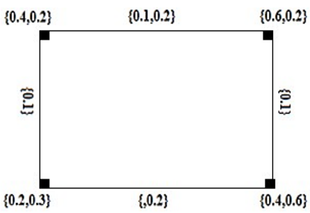

Fig. 2: HFG.

2.6 Definition [20]

A SVNS consists of some triplet of the form $\acute{A} = (\hat{S}, \tilde{I}, Ḑ)$ in set $\dot{X}$ where $\hat{S}$, $\tilde{I}$ and $Ḑ$ are functions from the set $\dot{X}$ to an element from unit interval $[0,1]$ with a restriction that $0 \leq \hat{S} + \tilde{I} + Ḑ \leq 3$. Further $\acute{A} = (\hat{S}, \tilde{I}, Ḑ)$ is called a single valued neutrosophic number (SVNN).

2.7 Definition [20]

Consider $\acute{A}_1 = (\hat{S}_1, \tilde{I}_1, Ḑ_1)$ and $\acute{A}_2 = (\hat{S}_2, \tilde{I}_2, Ḑ_2)$ be two SVNNs. Then

1. $\acute{A}_1 \oplus \acute{A}_2 = (\hat{S}_1 + \hat{S}_2 - \hat{S}_1\hat{S}_2, \tilde{I}_1\tilde{I}_2, Ḑ_1Ḑ_2)$
2. $\acute{A}_1 \otimes \acute{A}_2 = (\hat{S}_1\hat{S}_2, \tilde{I}_1 + \tilde{I}_2 - \tilde{I}_1\tilde{I}_2, Ḑ_1 + Ḑ_2 - Ḑ_1Ḑ_2)$
3. $\lambda\acute{A} = (1 - (1 - \hat{S}_1)^{\lambda}), \tilde{I}_1{}^{\lambda}, Ḑ_1{}^{\lambda})$
4. $\acute{A}_1{}^{\lambda} = (\hat{S}_1{}^{\lambda}, (1 - \tilde{I}_1)^{\lambda}, (1 - Ḑ_1)^{\lambda})$ where $\lambda > 0$

2.8 Definition [20]

The score, accuracy and certainty values of a SVNN $\acute{A}_1 = (\hat{S}_1, \tilde{I}_1, Ḑ_1)$ is of the form:

1. $s(\acute{A}_1) = \frac{2 + \hat{S}_1 - \tilde{I}_1 - Ḑ_1}{3}$
2. $a(\acute{A}_1) = \hat{S}_1 - Ḑ_1$
3. $c(\acute{A}_1) = \hat{S}_1$

2.9 Definition [20]

Consider $\acute{A}_1 = (\hat{S}_1, \tilde{I}_1, Ḑ_1)$ and $\acute{A}_2 = (\hat{S}_2, \tilde{I}_2, Ḑ_2)$ be the two SVNNs. Then:

1. $\acute{A}_1 < \acute{A}_2$ if $s(\acute{A}_1) < s(\acute{A}_2)$
2. $\acute{A}_1 > \acute{A}_2$ if $s(\acute{A}_1) > s(\acute{A}_2)$
3. $\acute{A}_1 = \acute{A}_2$ if $s(\acute{A}_1) = s(\acute{A}_2)$

2.10 Definition [44]

An SVNG is a duplet $Ģ = (\tilde{V}, \ddot{E})$ where $\tilde{V}$ is a set of nodes s.t. and $\hat{S}_1$, $\tilde{I}_1$ and $Ḑ_1$ are three mappings on $[0, 1]$ interval for $\tilde{v}_n \in \tilde{V}$ respectively with a restriction $0 \leq \hat{S}_1 + \tilde{I}_1 + Ḑ_1 \leq 3$ and $\ddot{E}$ is the set of edges where $e_i \in \tilde{V} \times \tilde{V}$ is based on three mappings $\hat{S}_2$, $\tilde{I}_2$ and $Ḑ_2$ defined as:

$\hat{S}_2(e_i) \leq \min[\hat{S}_1(\tilde{v}_i), \hat{S}_1(\tilde{v}_j)]$,
$\tilde{I}_2(e_i) \leq \max[\tilde{I}_1(\tilde{v}_i), \tilde{I}_1(\tilde{v}_j)]$, $Ḑ_2(e_i) \leq \max[Ḑ_1(\tilde{v}_i), Ḑ_1(\tilde{v}_j)]$ with a restriction that $0 \leq \hat{S}_2 + \tilde{I}_2 + Ḑ_2 \leq 3$.

2.11 Definition [47]

An SVNHFS consists of some triplet of the form $\tilde{N} = (\hat{S}, \tilde{I}, Ḑ)$ in set $\dot{X}$ where $\hat{S}$, $\tilde{I}$ and $Ḑ$ are sets of values from the set $\dot{X}$ to an element from unit interval $[0,1]$ with the restrictions that $0 \leq \alpha, \beta, \gamma \leq 1$ and $0 \leq \alpha^+, \beta^+, \gamma^+ \leq 3$ where $\alpha \in \hat{S}, \beta \in \tilde{I}, \gamma \in Ḑ$, $\alpha^+ \in \hat{S}^+ = \max \cup_{\alpha \in \hat{S}}$, $\beta^+ \in \tilde{I}^+ = \max \cup_{\beta \in \tilde{I}}$, and $\gamma^+ \in Ḑ^+ = \max \cup_{\gamma \in Ḑ}$. Further $\tilde{N} = (\hat{S}, \tilde{I}, Ḑ)$ is called a single valued neutrosophic hesitant fuzzy number (SVNHFN).

2.11.1 Example

Fig. 3 is an example of SVNG.

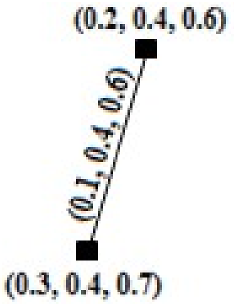

Fig. 3: SVNG.

3. Single Valued Neutrosophic Hesitant Fuzzy Graphs

In this section we will discuss the new concept of SVNHFG as a generalization of SVNG and HFG along with some important results and their properties.

3.1 Definition

A SVNHFG is a pair $Ģ = (\tilde{V}, \ddot{E})$ where $\tilde{V}$ is a set of nodes s.t. and $\hat{S}_1$, $\tilde{I}_1$ and $Ḑ_1$ are three HFNs that describe each $\tilde{v}_n \in \tilde{V}$ respectively with a restriction that $0 \leq \max \hat{S}_1 + \max \tilde{I}_1 + \max Ḑ_1 \leq 3$ and $\ddot{E}$ is the set of edges where $e_{\mathrm{i}} \in \tilde{V} \times \tilde{V}$ is based on three HFNs $\hat{S}_2$, $\tilde{I}_2$ and $Ḑ_2$ defined as:

$\hat{S}_2(e_{\mathrm{i}}) \leq \min[\hat{S}_1(\tilde{v}_{\mathrm{i}}), \hat{S}_1(\tilde{v}_{\hat{j}})]$,

$\tilde{I}_2(e_{\mathrm{i}}) \leq \max[\tilde{I}_1(\tilde{v}_{\mathrm{i}}), \tilde{I}_1(\tilde{v}_{\hat{j}})]$,

$Ḑ_2(e_{\mathrm{i}}) \leq \max[Ḑ_1(\tilde{v}_{\mathrm{i}}), Ḑ_1(\tilde{v}_{\hat{j}})]$ with a restriction that $0 \leq \max \hat{S}_2 + \max \tilde{I}_2 + \max Ḑ_2 \leq 3$.

3.1.1 Example

Fig. 4 is an example of SVNHFG.

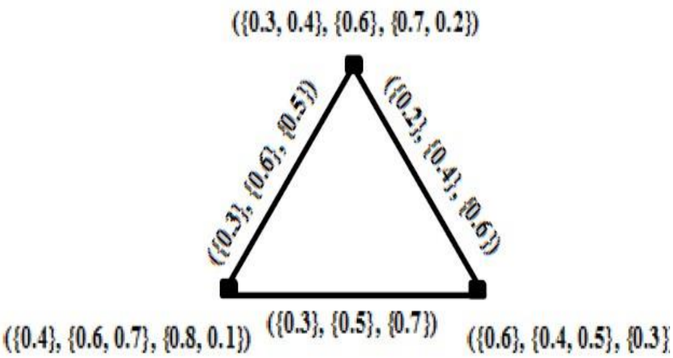

Fig. 4: SVNHFG.

3.2 Definition

A pair $S = (Ṿ', \ddot{E}')$ is a SVNHF subgraph of a SVNHFG $Ģ = (\tilde{V}, \ddot{E})$ if

1. $\hat{S}'_{1i} \leq \hat{S}_{1i}$, $\tilde{I}'_{1i} \leq \tilde{I}_{1i}$, $Ḑ'_{1i} \leq Ḑ_{1i}$
2. $\hat{S}'_{2ij} \leq \hat{S}_{2ij}$, $\tilde{I}'_{2ij} \leq \tilde{I}_{2ij}$, $Ḑ'_{2ij} \leq Ḑ_{2ij}$

   $\forall\, \mathrm{i}, \hat{j} = 1,2,3, \ldots n$.

3.3 Definition

A SVNHFG is said to be a complete SVNHFG if

$\hat{S}_{2ij} = \min(\hat{S}_{1i}, \hat{S}_{1j})$

$\tilde{I}_{2ij} = \max(\tilde{I}_{1i}, \tilde{I}_{1j})$

$Ḑ_{2ij} = \max(Ḑ_{1i}, Ḑ_{1j}) \;\forall\, e_{\mathrm{i}} \in \ddot{E}$.

3.3.1 Example

Fig. 5 is an example of complete SVNHF.

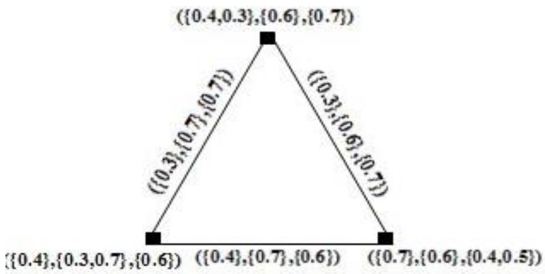

Fig. 5: Complete SVNHFG.

3.4 Definition

The complement of SVNHFG $Ģ = (\tilde{V}, \ddot{E})$ is a SVNHFG $\overline{Ģ} = (\overline{\tilde{V}}, \overline{\ddot{E}})$ where

1. $\overline{\tilde{V}} = \tilde{V}$ i.e. $\overline{\hat{S}_1}(\tilde{v}_{\mathrm{i}}) = \hat{S}_1(\tilde{v}_{\mathrm{i}})$, $\overline{\tilde{I}_1}(\tilde{v}_{\mathrm{i}}) = \tilde{I}_1(\tilde{v}_{\mathrm{i}})$ and

   $\overline{Ḑ_1}(\tilde{v}_{\mathrm{i}}) = Ḑ_1(\tilde{v}_{\mathrm{i}})$
2. $\overline{\hat{S}_2}(e_{\mathrm{i}}) = \min[\hat{S}_1(\tilde{v}_{\mathrm{i}}), \hat{S}_1(\tilde{v}_{\hat{j}})] - \hat{S}_2(e_{\mathrm{i}})$

$\overline{\tilde{I}_2}(e_{\mathrm{i}}) = \max[\tilde{I}_1(\tilde{v}_{\mathrm{i}}), \tilde{I}_1(\tilde{v}_{\hat{j}})] - \tilde{I}_2(e_{\mathrm{i}})$

$\overline{Ḑ_2}(e_{\mathrm{i}}) = \max[Ḑ_1(\tilde{v}_{\mathrm{i}}), Ḑ_1(\tilde{v}_{\hat{j}})] - Ḑ_2(e_{\mathrm{i}}) \;\forall\, \tilde{v}_{\mathrm{i}}, \tilde{v}_{\hat{j}} \in \tilde{V}$ an $e_{\mathrm{i}} \in \ddot{E}$.

3.4.1 Proposition

The complement of every SVNHFG is a SVNHFG.

3.4.2 Example

Fig. 6 shows SVNHFG, while Fig. 7 represents its complement.

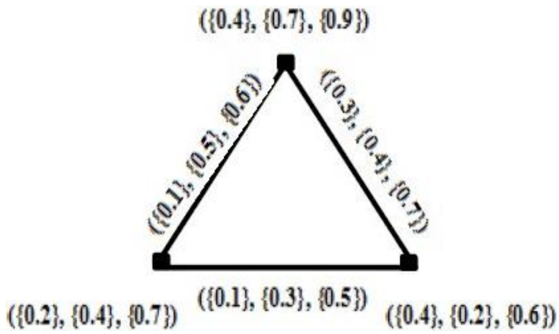

Fig. 6: SVNHFG.

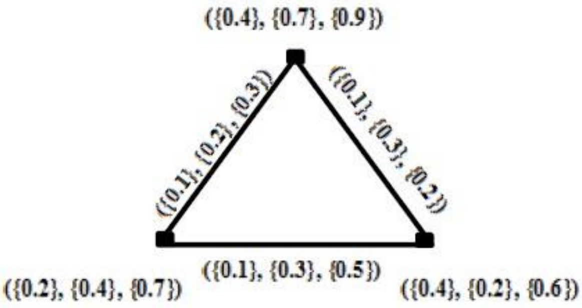

Fig. 7: Complement of SVNHFG.

3.5 Definition

The degree of vertex of a SVNHFG is defined by $đ(\tilde{v}) = (đ_{\hat{Ş}}(\tilde{v}), đ_{\tilde{I}}(\tilde{v}), đ_{Ḑ}(\tilde{v}))$ where,

$đ_{\hat{Ş}}(\tilde{v}) = \sum_{\tilde{v}\neq w}\hat{S}_2(\tilde{v}, w)$

$đ_{\tilde{I}}(\tilde{v}) = \sum_{\tilde{v}\neq w}\tilde{I}_2(\tilde{v}, w)$

$đ_{Ḑ}(\tilde{v}) = \sum_{\tilde{v}\neq w}Ḑ_2(\tilde{v}, w)$

where $đ_{\hat{Ş}}$, $đ_{\tilde{I}}$ and $đ_{Ḑ}$ denote the degree $\hat{S}$, $\tilde{I}$ and Ḑ, respectively.

3.6 Definition

The maximum degree of SVNHFG is defined by $\nabla(Ģ) = (\nabla_{\hat{Ş}}(Ģ), \nabla_{\tilde{I}}(Ģ), \nabla_{Ḑ}(Ģ))$ where, $\nabla_{\hat{Ş}}(Ģ) = \vee\, \{đ_{\hat{Ş}}(\tilde{v}); \tilde{v} \in \tilde{V}\}$

$\nabla_{\tilde{I}}(Ģ) = \vee\, \{đ_{\tilde{I}}(\tilde{v}); \tilde{v} \in \tilde{V}\}$ and

$\nabla_{Ḑ}(G) = \vee\, \{d_{Ḑ}(\tilde{v}); \tilde{v} \in \tilde{V}\}$ where $\nabla_{\hat{Ş}}$, $\nabla_{\tilde{I}}$ and $\nabla_{Ḑ}$ denotes the degree of maximum $\hat{S}$, $\tilde{I}$ and Ḑ, respectively.

The minimum degree of SVNHFG is defined by:

$\partial(Ģ) = (\partial_{\hat{Ş}}(Ģ), \partial_{\tilde{I}}(Ģ), \partial_{Ḑ}(Ģ))$ where

$\partial_{\hat{Ş}}(Ģ) = \wedge\, \{đ_{\hat{Ş}}(\tilde{v}); \tilde{v} \in \tilde{V}\}$

$\partial_{\tilde{I}}(Ģ) = \wedge\, \{đ_{\tilde{I}}(\tilde{v}); \tilde{v} \in \tilde{V}\}$

$\partial_{Ḑ}(Ģ) = \wedge\, \{đ_{FT}(\tilde{v}); \tilde{v} \in \tilde{V}\}$ where $\partial_{\hat{Ş}}$, $\partial_{\tilde{I}}$ and $\partial_{Ḑ}$ denote the degree of minimum $\hat{S}$, $\tilde{I}$ and Ḑ, respectively.

3.6.1 Example

Fig. 8 is a SVNHFG where degree of vertices is computed using definition (3.9).

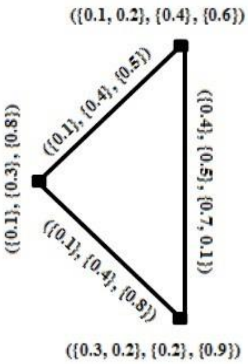

Fig. 8: Degree of SVNHFG.

$đ_{\hat{Ş}}(\tilde{v}_1) = 0.2$, $đ_{\tilde{I}}(\tilde{v}_1) = 0.7$, $đ_{Ḑ}(\tilde{v}_1) = 1.3$

$d(\tilde{v}_1) = (\{0.2\}, \{0.7\}, \{1.3\})$

$d(\tilde{v}_2) = (\{0.2\}, \{0.7\}, \{1.6\})$

$d(\tilde{v}_3) = (\{0.2\}, \{0.8\}, \{1.3\})$

$\partial(Ģ) = (\{0.2\}, \{0.7\}, \{1.3\})$

$\nabla(Ģ) = (\{0.2\}, \{0.8\}, \{1.6\})$

3.6.2 Proposition

In a SVNHFG $Ģ = (\tilde{V}, \ddot{E})$, the $\sum đ(\tilde{v}_{i}) = [2\sum_{\tilde{v}\neq w} đ(\tilde{v}, w)]$.

Proof:

Let $Ģ = (\tilde{V}, \ddot{E})$ be a SVNHFG.

Then

$\sum đ(\tilde{v}_{i}) = \sum[đ_{\hat{Ş}}(\tilde{v}_{i}), đ_{\tilde{I}}(\tilde{v}_{i}), đ_{Ḑ}(\tilde{v}_{i})]$

$= [\sum đ_{\hat{Ş}}(\tilde{v}_{i}), \sum đ_{\tilde{I}}(\tilde{v}_{i}), \sum đ_{Ḑ}(\tilde{v}_{i})]$ where $i = 1, 2, 3 \ldots n$.

$=[\sum_{i\neq j}\hat{S}_2(e_{i}), \sum_{i\neq j}\tilde{I}_2(e_{i}), \sum_{i\neq j}Ḑ_2(e_{i})]$

$=[\sum_{i\neq j}\hat{S}_2(e_{i}), \sum_{i\neq j}\tilde{I}_2(e_{i}), \sum_{i\neq j}Ḑ_2(e_{i})]$

$= [(\hat{S}_2(\tilde{v}_1, \tilde{v}_2), \tilde{I}_2(\tilde{v}_1, \tilde{v}_2), Ḑ(\tilde{v}_1, \tilde{v}_2)) + (\hat{S}_2(\tilde{v}_1, \tilde{v}_3), \tilde{I}_2(\tilde{v}_1, \tilde{v}_3), Ḑ_2(\tilde{v}_1, \tilde{v}_3)) + \cdots$
$+ \left(\hat{S}_2(\tilde{v}_1, \tilde{v}_n), \tilde{I}_2(\tilde{v}_1, \tilde{v}_n), Ḑ_2(\tilde{v}_1, \tilde{v}_n)\right)$
$+ \left(\hat{S}_2(\tilde{v}_2, \tilde{v}_1), \tilde{I}_2(\tilde{v}_2, \tilde{v}_1), Ḑ_2(\tilde{v}_2, \tilde{v}_1)\right)$
$+ \left(\hat{S}_2(\tilde{v}_2, \tilde{v}_3), \tilde{I}_2(\tilde{v}_2, \tilde{v}_3), Ḑ(\tilde{v}_2, \tilde{v}_3)\right) + \cdots$
$+ (\hat{S}_2(\tilde{v}_2, \tilde{v}_n), \tilde{I}_2(\tilde{v}_2, \tilde{v}_n), Ḑ_2(\tilde{v}_2, \tilde{v}_n)$
$+ \cdots \left(\hat{S}_2(\tilde{v}_n, \tilde{v}_1), \tilde{I}_2(\tilde{v}_n, \tilde{v}_1), Ḑ_2(\tilde{v}_n, \tilde{v}_1)\right)$
$+ \cdots (\hat{S}_2(\tilde{v}_n, \tilde{v}_{n-1}), \tilde{I}_2(\tilde{v}_n, \tilde{v}_{n-1}), Ḑ_2(\tilde{v}_n, \tilde{v}_{n-1}))]$

$= [2\sum_{i\neq j}\hat{S}_2(e_{i}), 2\sum_{i\neq j}\tilde{I}_2(e_{i}), 2\sum_{i\neq j}Ḑ_2(e_{i})]$

Hence proved.

3.6.3 Proposition

The maximum degree of any vertex in a SVNHFG with $n + 1$ vertex is n.

Proof:

Consider a SVNHFG. As $\max\,(\hat{S}_2(\tilde{v}_k, \tilde{v}_l)) = 1$ and maximum number of edges are 'n' of '$n + 1$' vertices. Then $d_{\hat{Ş}}(\tilde{v}_k) = n$ (In SVNHFG).

Similarly,

$d_{\tilde{I}}(\tilde{v}_n) = n$

$d_{Ḑ}(\tilde{v}_n) = n$,

Proved.

3.7 Definition

In a SVNHFG $Ģ = (\tilde{V}, \ddot{E})$ the path P is sequence of distinct vertices $\tilde{v}_1, \tilde{v}_2, \dots \tilde{v}_n$ such that either

$\hat{S}_2(\tilde{v}_{i-1}, \tilde{v}_i) > 0$

or

$\tilde{I}_2(\tilde{v}_{i-1}, \tilde{v}_i) > 0$

or

$Ḑ_2(\tilde{v}_{i-1}, \tilde{v}_i) > 0$

for $0 \leq i \leq 1$.

3.8 Definition

In a SVNHFG $G = (\tilde{V}, \ddot{E})$ the path P is sequence of distinct vertices $\tilde{v}_1, \tilde{v}_2, \dots \tilde{v}_n$, where $n \geq 1$ is called the length of the path. A single vertex $\tilde{v}_i$ is also a path but in this case the length of the path is $(\{0\}, \{0\}, \{0\})$.

3.9 Definition

A SVNHFG $Ģ = (\tilde{V}, \ddot{E})$ is connected if every two vertices have a SVNHF path between them otherwise disconnected.

3.10 Definition

In a SVNHFG $G = (\tilde{V}, \ddot{E})$, the consecutive pairs $(\tilde{v}_{i-1}, \tilde{v}_i)$ are called edges of path.

3.11 Definition

In a SVNHFG $Ģ = (\tilde{V}, \ddot{E})$, the path P is called a cycle if $\tilde{v}_0 = \tilde{v}_n$ where $n \geq 3$.

3.12 Definition

In a SVNHFG $G = (\tilde{V}, \ddot{E})$ if $\hat{S}_2 = \tilde{I}_2 = Ḑ_2 = 0$ then there does not exist any edge.

4. Algorithm for Finding Shortest Path in SVNHFG

To compute the shortest path from each node to its predecessor, an algorithm is proposed in this section. In real life problem this algorithm is useful to find the shortest path in a network.

Step 1:

Identify the first and final nodes of destination as $\tilde{v}_1$ and $\tilde{v}_n$.

Step 2:

Take $đ_1 = (\{0\}, \{1\}, \{1\})$ as there is no distance of node 1 from itself. Further, label the first node as $(\{0\}, \{1\}, \{1\}, -)$.

Step 3:

Find $đ_j = \min\{ đ_i \oplus đ_{ij}\}$. For $j = 2,3, \dots n$. Since the numbers are SVNHFNs so here we use mean value of function instead of using each value of function. That is $\hat{S}_1 = \frac{\hat{S}_{11} + \hat{S}_{12} \dots \hat{S}_{1n}}{n}$, where $n = 1, 2,3 \dots n$.

Step 4:

If the value of distance occurs against a unique $i = r$. Then j is marked as $[đ_j, r]$.

If the values of distance do not occur against a unique i. It represents more than one SVNHF paths from a node. So, to get the shortest among several paths, use the score function of SVNHFNs.

Step 5:

Let $[đ_n, k]$ is the label of the destination node then the shortest displacement between initial and final node is $đ_n$.

Step 6:

Since $[đ_n, k]$ is the label as destination node. So, for finding SVNHF shortest path from first node to last node, we check the label of node k. Let it be $[đ_n, l]$. Then we check the label of node l and so on. Repeat this process to obtain the initial node.

Step 7:

Hence, the SVNHF shortest path can be obtained by using step 6.

4.1 Example

Consider a network based on SVNHG given in Fig. 9 where the distance between the vertices is a SVNHFN. Using the proposed approach, the shortest path is computed as follows:

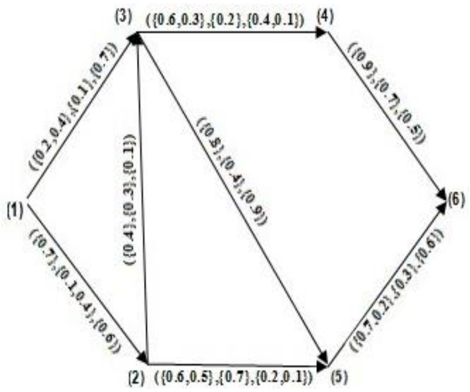

Fig. 9: SVNHF network.

The path between every two nodes is described in Table 1 using SVNHFNs.

Table 1: (SVNHF Network edges).

Edges	Distances
1 – 2	$(\{0.7\}, \{0.4, 0.1\}, \{0.6\})$
1 – 3	$(\{0.2, 0.4\}, \{0.1\}, \{0.7\})$
2 – 3	$(\{0.4\}, \{0.3\}, \{0.1\})$
2 – 5	$(\{0.6, 0.5\}, \{0.7\}, \{0.2, 0.1\})$
3 – 4	$(\{0.3, 0.6\}, \{0.2\}, \{0.4, 0.1\})$
3 – 5	$(\{0.8\}, \{0.4\}, \{0.9\})$
4 – 6	$(\{0.9\}, \{0.7\}, \{0.5\})$
5 – 6	$(\{0.7, 0.2\}, \{0.3\}, \{0.6\})$

Now we compute the shortest path by using the described algorithm as follows:

As the destination node is 6 so $n = 6$. Let $đ_1 = (\{0\},\{1\},\{1\})$ and mark the source node as $(\{0\},\{1\},\{1\},-)$ (say node 1), $đ_j$ can be found as follows.

Iteration 1:

Since the only predecessor of node 2 is 1, put $i = 1$ and $\hat{j} = 2$, by algorithm we get $đ_2$ as

$$đ_2 = \min\{\, đ_1 \oplus đ_{12}\}$$

$$= \min\big((\{0\},\{1\},\{1\}) \oplus (\{0.7\},\{0.4,0.1\},\{0.6\})\big)$$
$$= \min((\{0\},\{1\},\{1\}) \oplus ((0.7),(0.25),(0.6)))$$

$$= (\{0.7\},\{0.25\},\{0.6\})$$

Minimum occurs for one value of $i = 1$. So, vertex 2 is labeled as $[(\{0.7\},\{0.25\},\{0.6\})\}),-1]$

Iteration 2:

Since 1 and 2 are the predecessor of node 3 so

Put $i = 1, 2$ and $\mathrm{j} = 3$, by algorithm

$$đ_3 = \min\{\, đ_1 \oplus đ_{13}\,, đ_2 \oplus đ_{23}\}.$$

$$= min\,\{((\{0\},\{1\},\{1\}) \oplus (\{0.2,0.4\},\{0.1\},\{0.7\}),(\{0.7\},\{0.25\},\{0.6\}) \oplus (\{0.4\},\{0.3\},\{0.1\}\,).$$

$$= min(\{0.3\},\{0.1\},\{0.7\}),(\{0.82\},\{0.08\},\{0.06\}).$$

Using score function, we can get the minimum:

$s(\{0.3\},\{0.1\},\{0.7\}) = 0.5$ and

$s(\{0.82\},\{0.08\},\{0.06\}) = 0.89.$

So, the $đ_3 = (\{0.3\},\{0.1\},\{0.7\})$.

The minimum occurs for $i = 1$. So, vertex 3 is labeled as $[(\{0.3\},\{0.1\},\{0.7\}),1]$.

Iteration 3:

Node 3 is the predecessor of node 4 so put $i = 3$ and $j = 4$, by algorithm

$$đ_4 = \min\{\, đ_3 \oplus đ_{34}\}$$

$$= \min(\{0.3\},\{0.1\},\{0.7\}) \oplus$$

$$(\{0.3,0.6\},\{0.2\},\{0.4,0.1\})$$

$$= \min\{(\{0.3\},\{0.1\},\{0.7\}) \oplus$$

$$(\{0.45\},\{0.2\},\{0.25\}\} = (\{0.61\},\{0.02\},\{0.18\})$$

Minimum occurs for $i = 3$. So, vertex 4 is labeled as $[(\{0.61\},\{0.02\},\{0.18\}),3]$.

Iteration 4:

Node 2 and node 3 are predecessor of node 5 so put $i = 2, 3$ and $\mathrm{j} = 5$, by algorithm

$$đ_5 = \min\{\, đ_2 \oplus đ_{25}\,, đ_3 \oplus đ_{35}\}$$

$$= \min\big\{\{0.7\},\{0.25\},\{0.6\}\big) \oplus (\{0.6,0.5\},\{0.7\},\{0.2,0.1\}),$$

$$(\{0.3\},\{0.1\},\{0.7\}) \oplus (\{0.8\},\{0.4\},\{0.9\})$$

$$= \min\{(\{0.7\},\{0.25\},\{0.6\}) \oplus (\{0.55\},\{0.7\},\{0.15\}),$$

$$(\{0.3\},\{0.1\},\{0.7\}) \oplus (\{0.8\},\{0.4\},\{0.9\})\}$$

$$= \min \begin{pmatrix} \{0.88\},\{0.18\},\{0.09\}), \\ (\{0.86\},\{0.04\},\{0.63\}\} \end{pmatrix}$$

By score function we can get that,

$$đ_5 = (\{0.86\},\{0.04\},\{0.63\})$$

Minimum occurs for $i = 3$. So, vertex 5 is labeled as $[(\{0.86\},\{0.04\},\{0.63\}),3]$.

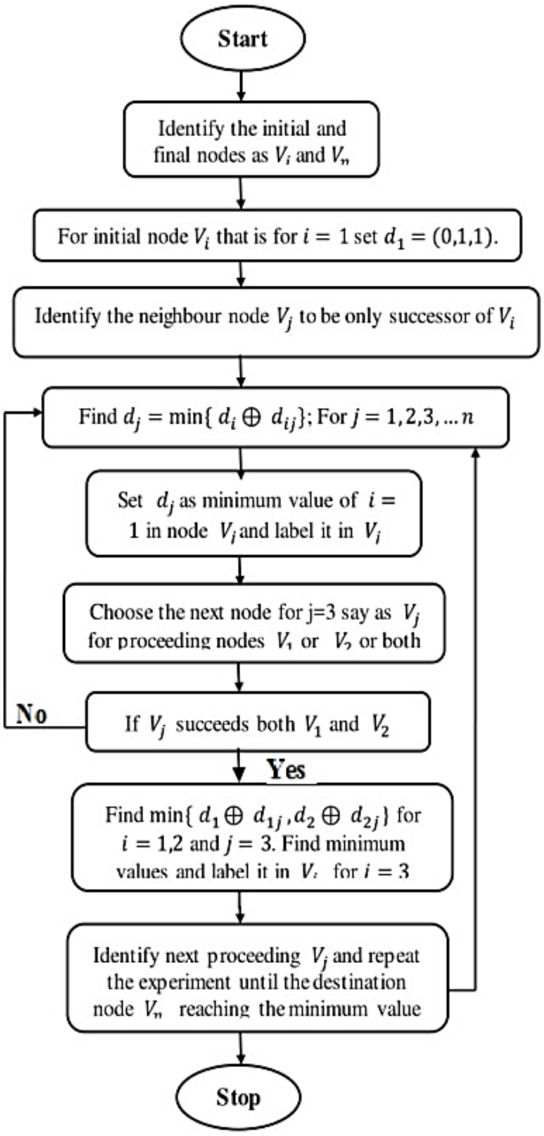

Fig. 10: (Flowchart of algorithm).

Iteration 5:

Nodes 4 and 5 are the predecessor of node 6 so put $i = 4, 5$ and $\mathrm{j} = 5$, by algorithm

$$đ_6 = \min\{\, đ_4 \oplus đ_{46}\,, đ_5 \oplus đ_{56}\}.$$

$= \min\{(\{0.61\},\{0.02\},\{0.18\}) \oplus (\{0.9\},\{0.7\},\{0.5\}),$

$= (\{0.86\},\{0.04\},\{0.63\}) \oplus (\{0.7,0.2\},\{0.3\},\{0.6\})\}.$

$= \min\{(\{0.61\},\{0.02\},\{0.18\}) \oplus (\{0.9\},\{0.7\},\{0.5\}),$

$= (\{0.86\},\{0.04\},\{0.63\}),\oplus (\{0.45\},\{0.3\},\{0.6\})\}.$

$= \min\{(\{0.96\},\{0.01\},\{0.09\}),$

$(\{0.92\},\{0.01\},\{0.38\})\}$

By score function we get that

$đ_6 = (\{0.92\},\{0.01\},\{0.38\}).$

Minimum occurs for $ị = 5$. So, vertex 6 is labeled as $[(\{0.92\},\{0.01\},\{0.38\}),5]$.

Since the destination point is $đ_6$. So, the shortest displacement from vertex one to six is provided as:

$(\{0.92\},\{0.01\},\{0.38\})$.

The shortest way can be determined as follows:

Node 6 is labelled as $[(\{0.92\},\{0.01\},\{0.38\}),5]$

Node 5 is labelled as $[(\{0.86\},\{0.04\},\{0.63\}),3]$

Node 3 is labelled as $[(\{0.61\},\{0.02\},\{0.18\}),1]$.

Hence, the shortest way is $1 \longrightarrow 3 \longrightarrow 5 \longrightarrow 6$

with the SVNHF value of distance being $(\{0.92\},\{0.01\},\{0.38\})$.

In Fig. 10 the dotted line represents the shortest path and Table 2 provides the path of different nodes.

Table 2: Shortest path.

Nodes No. (j)	d_i	Shortest path from 1st node to jth node
2	$(\{0.7\},\{0.25\},\{0.6\})$	$1 \longrightarrow 2$
3	$(\{0.3\},\{0.1\},\{0.7\})$	$1 \longrightarrow 3$
4	$(\{0.61\},\{0.02\},\{0.18\})$	$1 \longrightarrow 3 \longrightarrow 4$
5	$(\{0.86\},\{0.04\},\{0.63\})$	$1 \longrightarrow 3 \longrightarrow 5$
6	$(\{0.92,\}\{0.01\},\{0.38\})$.	$1 \longrightarrow 3 \longrightarrow 5 \longrightarrow 6$

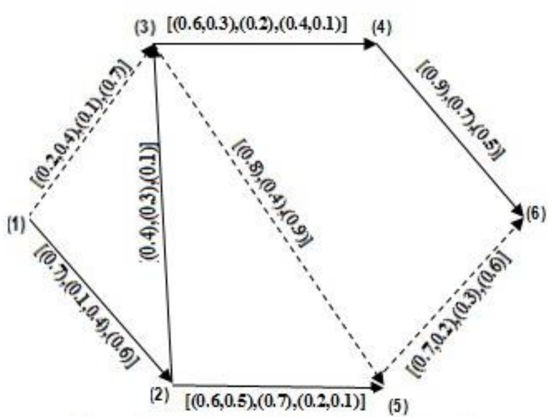

Fig. 11: Shortest path network of SVNHFNs.

5. Conclusions

In this article, an algorithm for computing the shortest path in a network having nodes in the form of SVNHFNs is developed. The key findings of the present study are as under:

a. The concept of SVNHFG is introduced which is an extension of neutrosophic graph and hesitant fuzzy graph.

b. Terms like complement, subgraphs, degree and path in SVNHFGs are defined and supported with examples.

c. An algorithm for computing shortest path is proposed which is further demonstrated by a flowchart.

d. A numerical example is solved using the proposed algorithm where the shortest path in a network is computed among all possible paths.

e. It is observed that the results obtained in this study are compatible with those presented by Broumi et al. [9-13].

In future, our aim is to extend this work to produce further interesting graph theoretic results in the environment of SVNHFGs and to utilize them in multi-attribute decision making and supply chain management problems.

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