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INVESTIGATING TARGET TRACKING PROBLEM USING A SIMPLE DE-CORRELATION SCHEME

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The target tracking problem for radar by using extended Kalman filter is investigated in this research work. First of all, we have investigated the problem of target tracking in Cartesian coordinates with polar measurements. To compensate for the non-linearity existing due to Cartesian to polar coordinate transformation, the extended Kalman filter is employed to get the state estimate of the target. In most of the modern radars, the measurement frequency is much higher thus, causing correlation in the measurement errors. If this correlation is not considered in the measurement model then the tracking performance will certainly degrade. The Kalman filter equations need to be modified while taking correlated noise into account. There are different techniques available for de-correlation of colored noise. A simple de-correlation scheme is proposed for tracking target which is also undergoing maneuver due to atmospheric turbulence. Simulation results show that significant improvement in tracking performance is obtained considering noise correlation.

Keywords: Extended Kalman Filter (EKF), De-correlation techniques, Noise correlation

1. Introduction

Target tracking trajectory estimation has found wide applications in both military and commercial field such as global positioning system, inertial navigation, guidance and control, missile guidance system, fire control system, underwater target tracking system. Many problems have been resolved whereas engineers are still confronted with new and diversified challenges.

Kalman Filter equations are the basis of the design of most of the tracking filters in which process and measurement noise is presumed white [1]. For white process and measurement noise conventional Kalman filter is frequently used for tracking non-maneuvering targets. However, when the target is maneuvering then the traditional Kalman filter needs to be modified to maintain the tracking performance. Several approaches are available to tackle this problem [2-8].

In the real world the measurement noise is not white. It is auto-correlated within a bandwidth of typically a few hertz. For instance, target scintillation (glint) at sufficiently closed ranges gives rise to range and angle measurement error to have a finite bandwidth on the order of several Hertz [9]. Moreover, target velocity variations and radar frequency instability also causes correlated measurement errors. Treating the correlation noise as a first order Markov process, the de-correlation of noise can be carried out in such a way that the (modified) Kalman filter is effective after de-correlation.

A possible approach to avert the effect of correlated noise is state augmentation technique in which target state vector is augmented with the state vector of the error model and resultantly the system is devoid of any measurement noise [10]. But this approach is sometimes problematic due to ill-conditioning of covariance matrix of state estimate. A solution to this problem is to use measurement differencing approach [11]. A similar approach has also been used in [12-16]. Rogers modeled colored noise as a first order AR process [9]. In this work we use Bryson and Henrikson approach for modeling of colored noise to de-correlate it [11]. Since radars use polar coordinates for position measurements therefore using above de-correlation scheme, this research work is extended for non-linear systems in which a popular form of Kalman filter called Extended Kalman filter (EKF) is employed to compensate for non-linearties occurring due to Cartesian to polar transformation. The target is also perturbed by atmospheric turbulence giving rise to a little maneuver [17] and then using the values of

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different coefficient matrices as given in the appreciable work of Singer [18], a considerable improvement in performance is obtained.

In section 2 we have introduced the dynamic model in addition to the radar polar coordinate transformation of measurements. Section 3 deals with the modeling of colored noise/correlated noise and its decorrelation for its use in the Extended Kalman filter. Section 4 presents the simulation results along with performance comparison after using the algorithm in EKF.

2. System Model

Singer derived the dynamic model of the maneuvering target and treated the maneuver variable (acceleration) as a first order autoregressive process [12].

$$\dot{a}(t) = \lambda a(t) + w(t) \tag{1}$$

Where λ is the correlation constant and *w(t)* is the noise process. The target motion and noisy measurement model of radar is given as [2]

$$X(k+1) = ØX(k) + Gw(k) \tag{2}$$

$$Z(k) = HX(k) + v(k) \tag{3}$$

Where X(k) and Z(k) are target state and measurement vector, respectively. w(k) and v(k) are white process and measurement noise, respectively. The computations of Ø, G, H and Q are taken same as given in reference [12], where Q is called transition matrix, G is the noise margin, H is Jacobian matrix and Q is covariance.

When the measurement frequency is much lower than the error bandwidth, the successive measurement errors are uncorrelated. While in most of the modern radars the correlation cannot be ignored because of very high measurement frequency.

Since radars use polar coordinates therefore, in this work extended Kalman filter has been used due to nonlinearity existing because of Cartesian to polar transformation [13]. State vector on the Cartesian coordinates is given as below:

$$X(k) = \begin{bmatrix} x \\ \dot{x} \\ y \\ \dot{y} \end{bmatrix} \tag{4}$$

$$Z(k) = \begin{bmatrix} r \\ \theta \end{bmatrix} \tag{5}$$

Where (x, y) target are positions and $(\dot{x}, \dot{y})$ are the corresponding velocities. Nonlinear relationship for Z (k) is given as under**:**

$$Z(k)=h\{x(k)\}+v(k)= \begin{bmatrix} \sqrt{x(k)^2 + y(k)^2} \\ \tan^{-1} y(k)/x(k) \end{bmatrix} + \begin{bmatrix} v_r \\ v_\theta \end{bmatrix}. \tag{6}$$

Where V_r, v_θ are zero mean white Gaussian distribution. If any of the system or measurement relation is non-linear the expansion is carried out by Taylor series.

$$h\ (x(k)) \approx h(X(k/k\text{-}1) + Hk\ (X(k) - X(k/k\text{-}1) \tag{7}$$

$$H^k = \frac{\partial h(x)}{\partial x}\Big|_{x=X(k/k-1)} = \begin{pmatrix} x/r & y/r \\ -y/r^2 & x/r^2 \end{pmatrix} \tag{8}$$

Where H^k is Jacobean matrix of h(.) measurement function at X(k/k-1) and h(.) is the Cartesian to polar coordinate transformation. Filter relations are given as below:

State estimate:

$$X(k+1/k) = \Phi X(k/k) \tag{9}$$

Innovation is given as:

$$Inn(k+1) = Z(k+1) - h[X(k/k-1)] \tag{10}$$

Covariance of state error:

$$P(k+1/k) = \Phi P(k/k)\Phi' + GQG' \tag{11}$$

Covariance of innovation:

$$S(k+1) = H_k P(k+1/k) H' + R \tag{12}$$

Gain of the filter:

$$K(k+1) = P(k+1/k) H'_k S(k+1)^{-1} \tag{13}$$

Estimate of covariance:

$$P(k+1/k+1) = [I - k(k+1)H_k]P(k+1/k)] \tag{14}$$

Again updated state estimate:

$$X(k+1/k+1) = X(k+1/k) + K(k+1) inn(k+1) \tag{15}$$

3. Modeling of Colored Noise and its De-correlation

Consider the system given by equations:

$$\begin{aligned} x_k &= F_{k-1} x_{k-1} + w_{k-1} \\ y_k &= H_k x_k + v_k \end{aligned} \tag{16}$$

We assume that the correlated noise can be modeled as a first order Markov process given by

$$\dot{v}(t) = -\beta v(t) + w(t) \tag{17}$$

The discrete version is:

$$v_k = \psi_{k-1} v_{k-1} + \varsigma_{k-1} \tag{18}$$

Where ς_k is the white noise. To de-correlate the colored measurement noise, a pseudo measurement y'_k is generated. By taking ψ as preset value of color, the noise measurement equation can be re-derived as given:

$$y'_{k-1} = y_k - \psi_{k-1} y_{k-1}$$

$$y'_{k-1} = (H_k x_k + v_k) - \psi_{k-1}(H_{k-1} x_{k-1} + v_{k-1})$$

$$= H_k (F_{k-1} x_{k-1} + w_{k-1}) + v_k - \psi_{k-1}(H_{k-1} x_{k-1} + v_{k-1})$$

$$= (H_k F_{k-1} - \psi_{k-1} H_{k-1}) x_{k-1} + (H_k w_{k-1} + \varsigma_{k-1})$$

$$= H'_{k-1} x_{k-1} + v'_{k-1} \tag{19}$$

We have now a new measurement equation that has a measurement matrix H'_{k-1} and measurement noise v'_{k-1}. Our new system ready to be used in the Kalman filter can be formulated as:-

$$x_k = F_{k-1} x_{k-1} + w_{k-1} \tag{20}$$

$$y'_k = H'_k x_k + v'_k \tag{21}$$

4. Simulation Results

A Matlab/Simulink® model with 100 runs has been used to demonstrate the tracking performance. As shown in the flow chart of Figure 1, the horizontal as well as vertical accelerations are integrated to get corresponding velocities which on further integration gives position of the target in two dimensional planes. Since radars use polar coordinates thus it entails coordinate conversion from Cartesian to polar coordinates. The polar coordinates of position measurements are fed into the Kalman filter alongwith colored measurement noise to get the position estimates and residual error. For designing of colored noise Rogers modeling of colored noise is used. Moreover, the use of polar coordinates make the system nonlinear thus the measurement matrix is first linearized using the Jacobean matrix. The initial parameters are chosen and the coefficient matrices Ø, G, H and calculation of Q are same as mentioned in reference [12] with α=0.5 and target speed 450 m/sec. The de-correlated system has sufficiently satisfactory performance giving vertical and horizontal position estimates of the target.

The performance of the proposed algorithm has been evaluated by means of simulation analysis. The designed model simulates the target dynamics; generates measurement as per the accepted noise model; implements the algorithm of interest and thereafter executes the posterior statistical processing of data. Whenever color noise is added to the system, the performance of the system is degraded unless a de-correlation scheme is used to circumvent colored noise. The results show the improvement in performance of the system after using a *simple proposed de-correlation scheme* while using extended Kalman Filter.

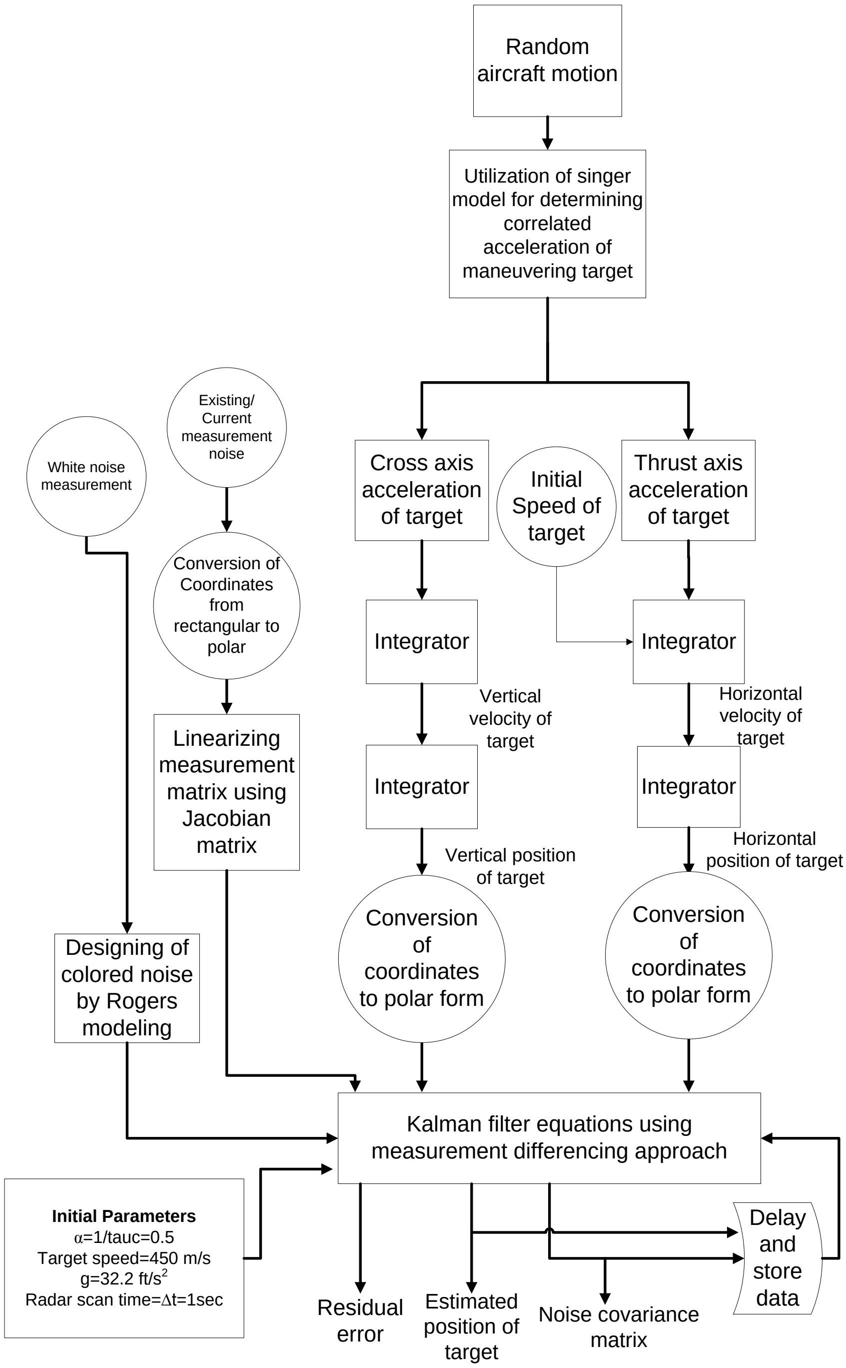

Figure 1. Flow chart for the designed algorithm.

4.1. *Mathematical Model of the Target*

The physical characteristics of the target whether it is moving at constant velocity or it is undergoing some maneuver (acceleration) are important for consideration. The mathematical model of target is shown in Figure 2. In this model a target is generated which is also undergoing a little maneuver due to atmospheric turbulence. The atmospheric turbulence may provide correlated acceleration for 1 or 2 seconds. As correlation coefficient α is the reciprocal of maneuver time constant therefore α=0.5.

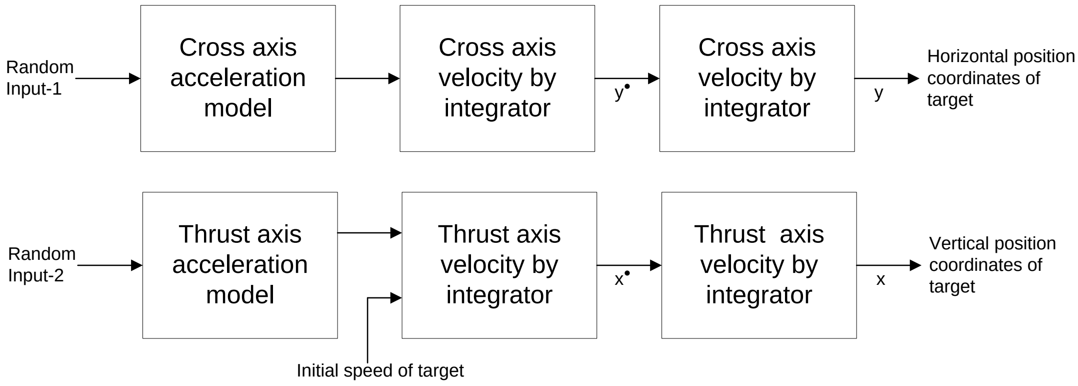

Figure 2. Mathematical model of target.

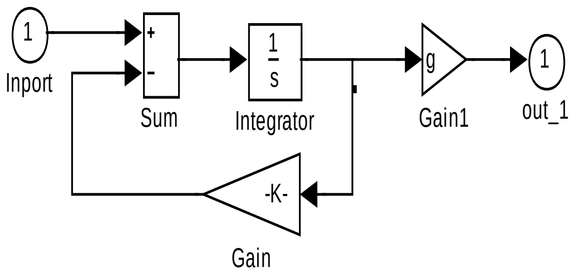

Figure 3. Block diagram of maneuvering model of target.

In Figure 3, the target is undergoing correlated acceleration for a period of 1 or 2 seconds. The value of K=α=0.5 is the reciprocal of maneuver time constant (tauc). If the correlation is increased then the performance will start degrading. For example if we take k=1/17 (i.e. tauc=17 seconds) then the target will be maneuvering abruptly more than just because of atmospheric turbulence. Performance will highly be degraded as shown in the Figures 4 and 5.

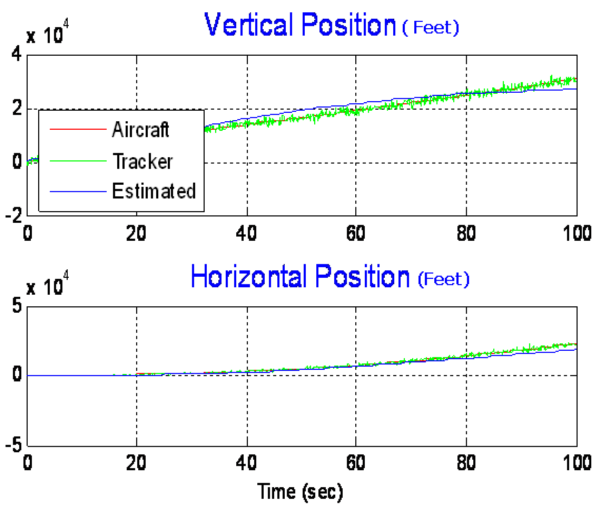

Figure 4. Maneuvering target performance (tauc=17 seconds).

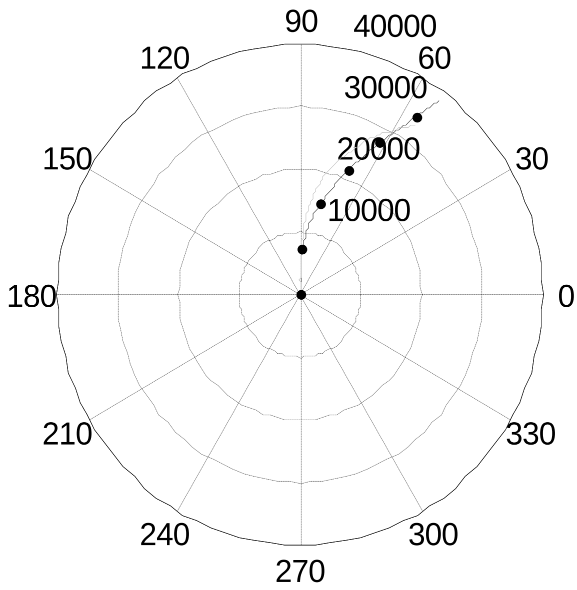

Figure 5. Target showing abrupt maneuver (tauc=17 seconds).

The actual trajectory (line with circular dots) and the estimated trajectory (solid line) in the Fig. 5 are showing considerable error more than 10% for a target which is undergoing abrupt maneuver.

When the value of maneuver time constant (tauc) is taken to be 1 or 2 seconds indicating the maneuver as due to atmospheric turbulence then the results are far better as shown in Figures 6 and 7.

4.2. Filtering Algorithm

The filtering algorithm is shown in Figure 8. Since radars use polar coordinates for measuring position therefore, measurements are first converted to polar coordinates from the Cartesian one. The polar measurements from radar are fed

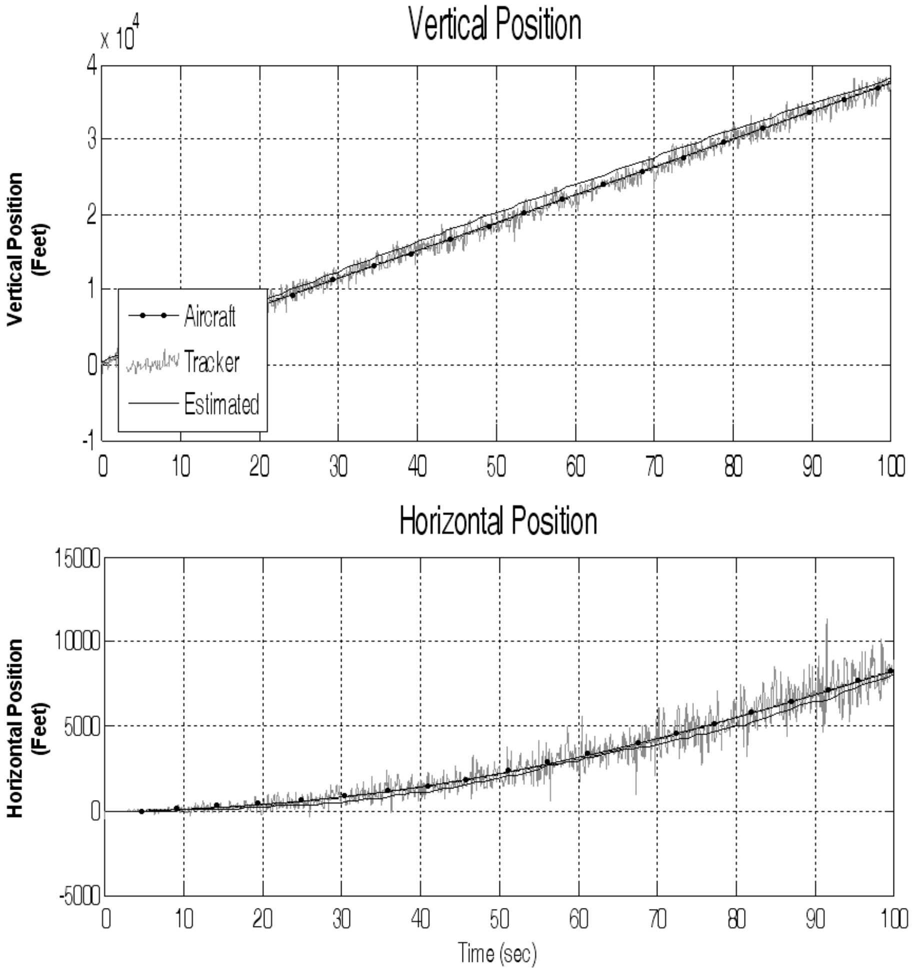

Figure 6. Target maneuvering due to atmospheric turbulence (tauc=2 sec).

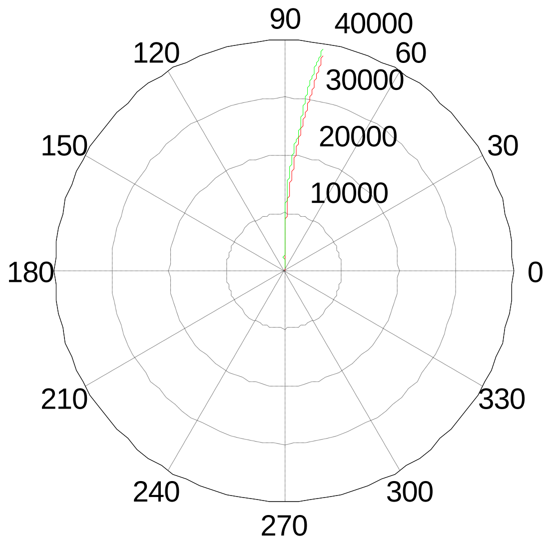

Figure 7. Maneuver due to atmospheric turbulence.

into the Extended Kalman filter which estimates the current and future positions of the target. The Extended Kalman filter also compensates for the non linearity existing due to Cartesian to polar coordinate transformation. The output of the filtering process is the residual in range and estimated positions in vertical and horizontal directions.

4.3. External Disturbances

4.3.1. Addition of White Noise

It is first assumed that all the noises acting on the system are white and Gaussian distributed. But this is unrealistic assumption since in reality the noises are not white. For example in the case of radars, the measurement frequency is quite high so correlation in measurement errors cannot be ignored without tracking performance deterioration.

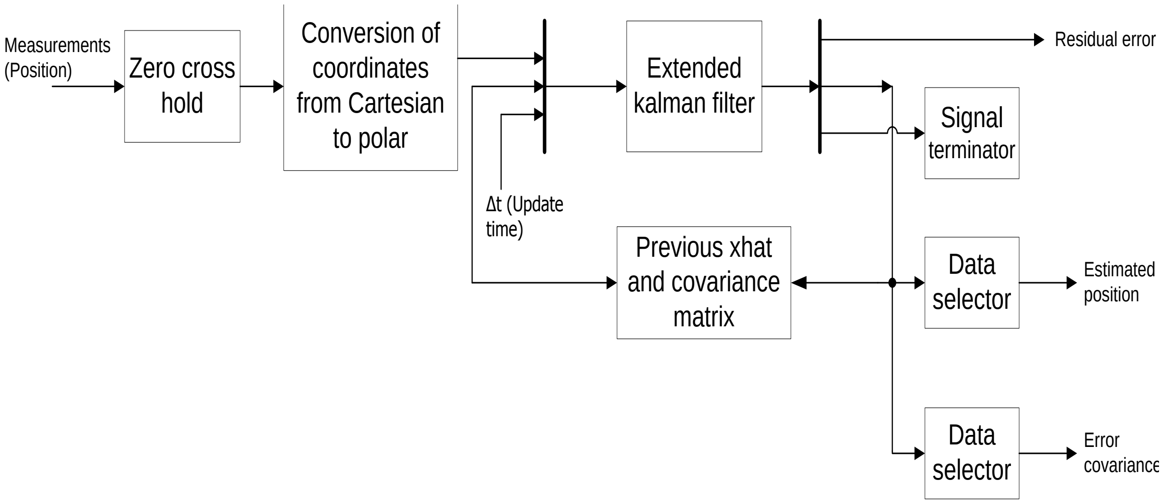

Figure 8. Data processing algorithm.

The results obtained by considering white noise are shown in Figure 9. These results are obtained by ignoring correlation in measurement errors.

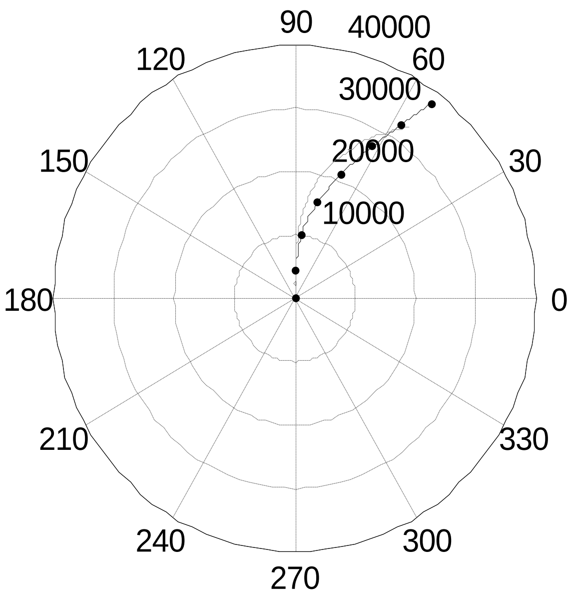

Figure 9. Actual (lines with circular dots) versus estimated (solid line) trajectory by considering white noise (ignoring correlation).

4.3.2. Addition of Colored Noise

To make the system more realistic, colored noise is added to the system as shown in Figure 10. Whenever a colored noise is used; the Kalman filter equations require modifications because Kalman filter assumes white noise. So the colored noise first needs to be de-correlated so that the conventional Kalman filter could be employed for processing of data. The performance of the system is analyzed after de-correlating colored noise by using a suitable de-correlation scheme. As shown in Figure 11, the modeling of colored noise is done by using the approach of Rogers [9]. The feedback gain is the value of color (ψ) which varies from 0 to 1 and from model to model. In this research work the value of ψ is taken to be 0.6. As the value of ψ increases there is a very slight degradation of performance. However the error due to degradation of performance is not greater than 2%. It means that the system is still suitable for real time applications.

The target is generated as per the target model specified and then measurements are made correlated in time to build a more realistic model and thereby improving the overall performance of the system. The performance of the algorithm has been evaluated by means of simulation analysis. The designed model performs following tasks; simulates the target dynamics; generates measurement as per the accepted noise model; implements the algorithm of interest and finally executes the posterior statistical processing of data. Whenever color noise is added to the system the performance of the system is degraded unless a de-correlation scheme is used to circumvent colored noise. The results in the Figures 12-14 have shown significant improvements in the performance of the system after using a simple proposed de-correlation scheme while using extended Kalman Filter.

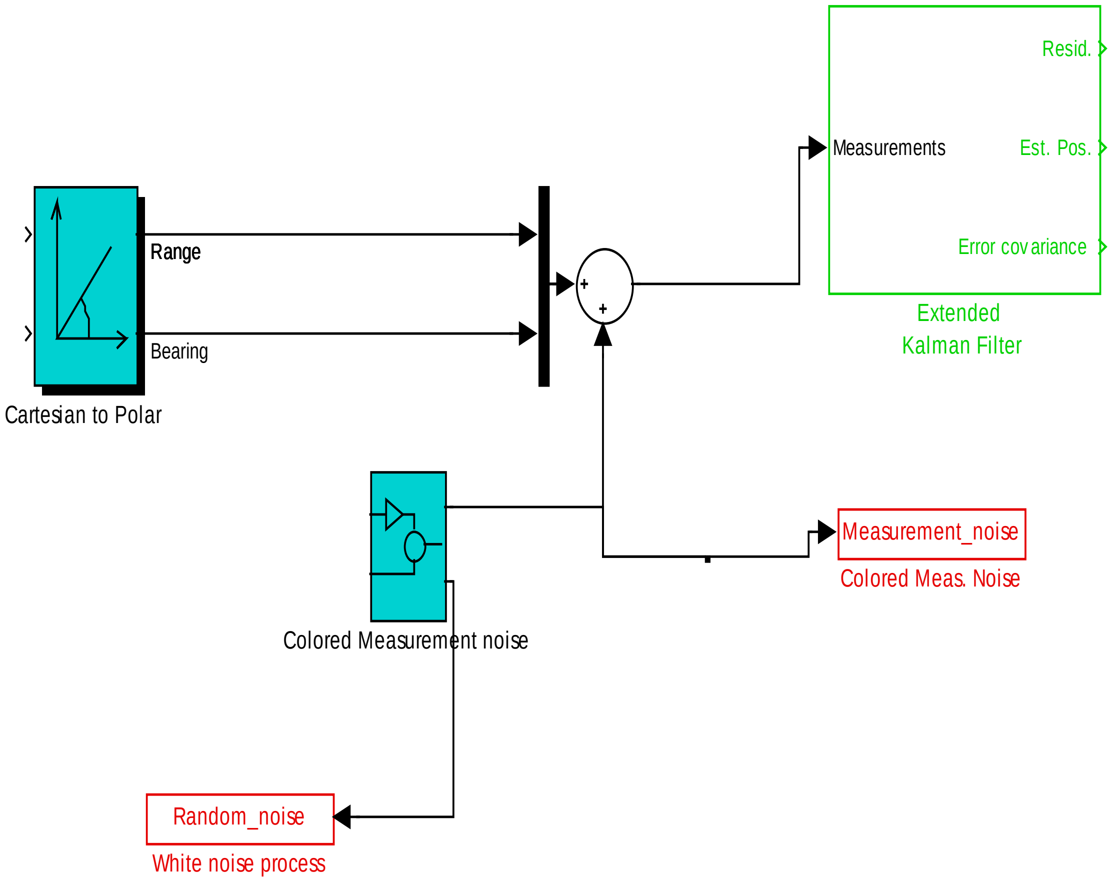

Figure 10. Colored noise model.

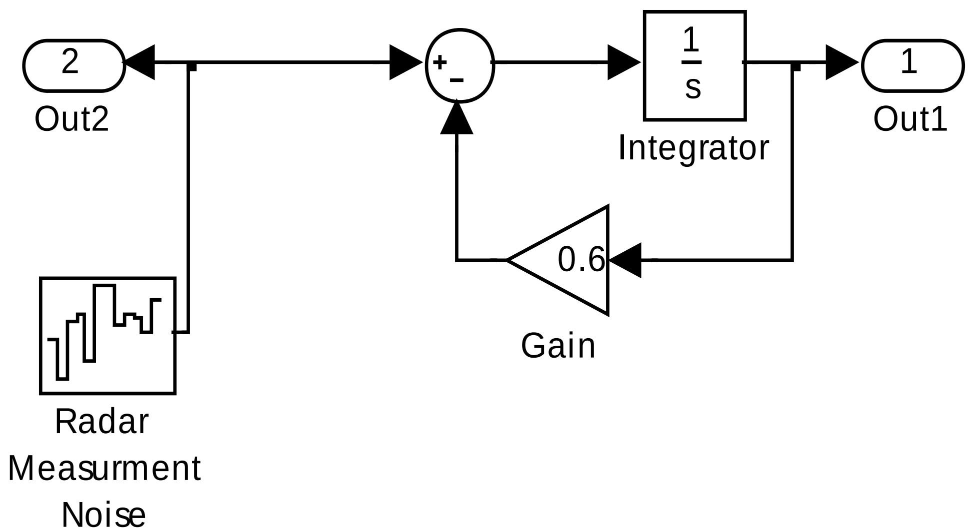

Figure 11. Block diagram of model of colored noise.

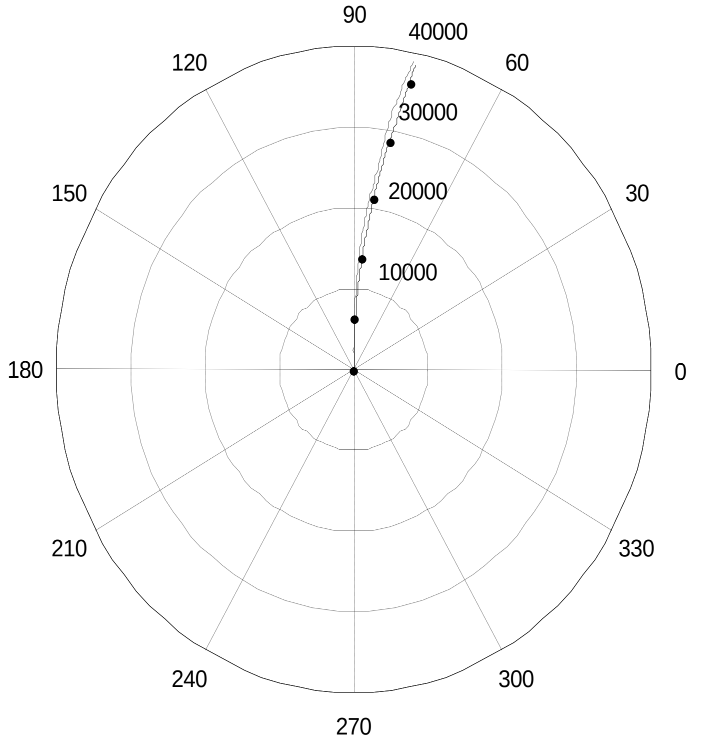

Figure 12. Actual versus estimated trajectory.

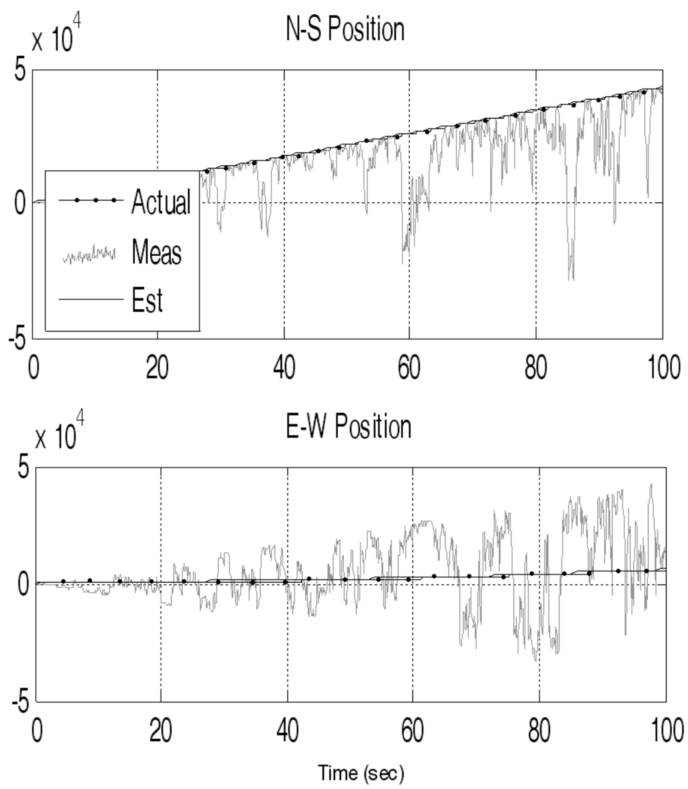

Figure 13. Horizontal and vertical estimates of position.

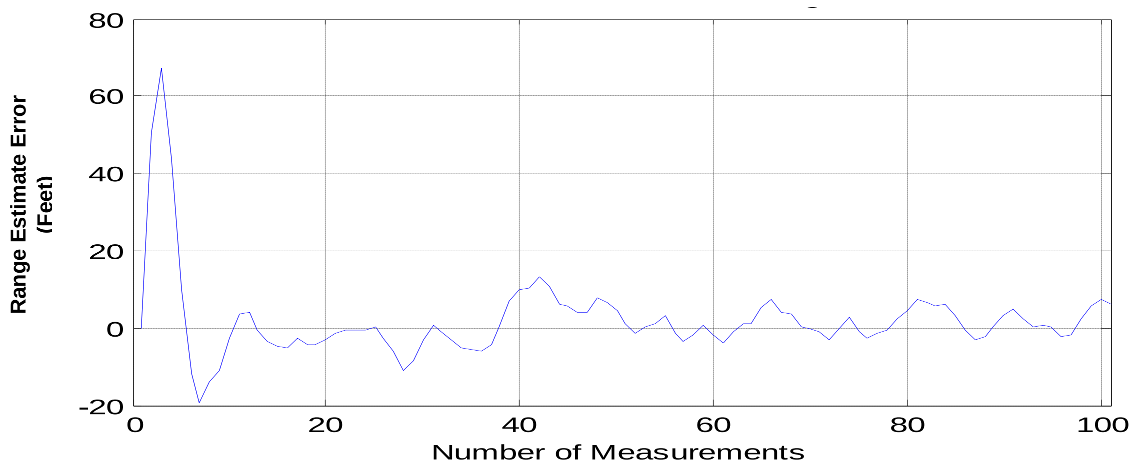

Figure 14. Residual error in range.

As it can be seen from Figure 14, that error has been sufficiently reduced to almost 2% which is quite suitable for tracking real time systems. The suitable de-correlation algorithm is quite effective and the overall performance of the system is much better than the previous results where white noise was being added for real time systems. Table 1 below shows the comparison of white and colored noise models results for position error considering *proposed simple de-correlation scheme*.

Table 1. The comparison of white and colored noise models.

Target	Noise	Position error
Non-Manuevering	Undercorrelated	5%
	Decorrelated	1%
Manuevering	Undercorrelated	10%
	Decorrelated	2%

4. Conclusions

In this research work we have utilized extended Kalman filter to investigate the problem of target tracking by radar. The problem considered is that of estimating the state variables of a linear dynamic system based on the measurements containing the contents of colored noise. The measurement frequency in most of the modern radars is quite higher than the error bandwidth, therefore, the errors in the successive measurements cannot be treated as white noise without tracking performance deterioration. Whenever the colored noise is added to the system the conventional Kalman filter equations need to be modified by using different de-correlation techniques. A simple proposed de-correlation scheme is employed to circumvent the effect of colored noise for a target which is also made to maneuver by atmospheric turbulence. The computer simulations show that the system performance can be improved significantly if noise correlation is considered.

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