

A HYBRID PI CONTROL SCHEME FOR AIRSHIP HOVERING

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Airship provides us many attractive applications in aerospace industry including transportation of heavy payloads, tourism, emergency management, communication, hover and vision based applications. Hovering control of airship has many utilizations in different engineering fields. However, it is a difficult problem to sustain the hover condition maintaining controllability. So far, different solutions have been proposed in literature but most of them are difficult in analysis and implementation. In this paper, we have presented a simple and efficient scheme to design a multi input multi output hybrid PI control scheme for airship. It can maintain stability of the plant by rejecting disturbance inputs to ensure robustness. A control scheme based on feedback theory is proposed that uses principles of optimality with integral action for hovering applications. Simulations are carried out in MTALAB for examining the proposed control scheme for hovering in different wind conditions. Comparison of the technique with an existing scheme is performed, describing the effectiveness of control scheme.

Keywords: Airship, Dynamic modeling, Hovering, Control law, Robustness, MIMO hybrid PI control scheme.

1. Introduction

Vehicles like airships present unique and promising solutions for robotics and aviation industry that involves a long-endurance and autonomous operation. Having their lift with buoyancy force, these vehicles require much less power than traditional aircraft. With the help of renewable energy sources, a controlled flight can be maintained for an indefinite period. In radio communication and wireless networks, the use of airship as a station, keeping its position fixed, can provide link similar to the satellite. Major applications of LTA include hovering or roving, surveillance, telecommunication, remote-sensing, etc. Operating the vehicle at 60~70 thousand feet height has several more attractive features, making it a promising solution for both government programs and commercial ventures where it can maintain a geostationary location closer to the earth. In order to achieve these objectives, a robust guidance and control system capable of auto-piloting under wide range of atmospheric and wind conditions is required. First, successful design of such a system requires an accurate model of airship dynamics. A 6 degrees-of-freedom dynamic model of a non-rigid airship is used as a basis for

design of a controller. The model includes all inertial, dynamic, aerodynamic, gravitational, buoyancy and propulsion forces. It is capable of explaining the stability and control performance alongwith the flight modes of the airship.

For the observation oriented applications, airship may be able to sustain stationary flight state (hover flight) independent of the atmospheric disturbances indicated [1]. Recently, many control techniques have been applied to solve the hovering control problem for different types of airships. Hovering flight in turbulent wind cases was initially discussed [2]. Whereas, stability and control issues for VTOL capable airships in hovering flight were investigated [3]. Using the decoupled formulation, design of controller for hovering application using image feedback with two control loops was realized [4]. An image-based visual servoing in a PD error feedback scheme was used for automatic hovering of an outdoor autonomous airship [5]. This technique was used for hover control [6]. The station keeping of large high altitude airship is suggested [7]. A back stepping control algorithm for airship hovering with input saturations is proposed [8, 9].

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Nomenclature

A_r	= 6x1 Aerodynamics Vector
A_{long}	= Linearised system matrix for longitudinal motion
A_{lat}	= Linearised system matrix for lateral motion
a	= Linearised system matrix of plant "airship"
a^*	= Modified system matrix of the airship
A	= Linearised system matrix of the augmented system
B_{long}	= Linearised input matrix for longitudinal motion
B_{lat}	= Linearised input matrix for lateral motion
B	= Linearised input matrix of the augmented system
b	= Linearised input matrix of airship
c	= Linearised output matrix of airship
e	= Error vector between reference input and output vectors of airship
F_d	= 6x1 Dynamic Vector
G	= 6x1 Gravitational Vector
K_{pol}	= Feedback gain matrix for pole-placement
K_{opt}	= Feedback gain matrix for optimal control
M	= 6x6 Mass Matrix
N, E, h	= North, East and height positions in earth axis system
n	= No of states matrix of the system
\bar{n}	= No of states matrix of the augmented system
P	= 6x1 Propulsion Vector
p, q, r	= Airship angular velocities in roll, pitch and yaw respectively (rad/s)
Q	= Positive-semi definite constant coefficient matrix in optimal control
R	= Positive definite constant coefficient matrix in optimal control
r_i	= Reference input vector
u	= Control input to the plant "airship"
u_x, v, w	= Airship linear velocities along X, Y and Z directions in its, 'Body' axes system in m/s

u_{long}	= Input vector of the longitudinal motion
u_{lat}	= Input vector of the lateral motion
x	= State vector of the airship dynamics
x_{long}	= State vector of the longitudinal motion
x_{lat}	= State vector of the lateral motion
X	= Augmented state vector of airship dynamics
y	= Linearised output vector of the airship dynamics
ϕ, θ, φ	= Vehicle's Euler angles in Earth Inertial system
δ_{er}	= Deflection of right elevator
δ_{el}	= Deflection of left elevator
δ_{tx}	= Thrust perturbations along X-axis
δ_{tz}	= Thrust perturbations along Z-axis
δ_{rt}	= Deflection of top rudder
δ_{rb}	= Deflection of bottom rudder
δ_{ty}	= Thrust perturbations along Y-axis

This paper presents a simple but efficient approach for the stabilization and control of airship hover flight. It is evident from [10], airship may exhibit oscillatory behavior in some of its modes during hovering flight. Keeping in view of vehicle dynamics, a two fold controlling strategy is introduced. Inner control loop places the poles of system at stable location to avoid such oscillations. Outer loop performs regulation action in order to track hovering reference. The preset position and attitude reference tracking is performed in an optimal and robust manner, ensuring its asymptotic stability.

2. Dynamic Modeling of the Airship

The concept of mathematical model used in this flight simulation model is basically suggested by [10, 11]. It is a full six-degree-of-freedom (6DOF) non-linear mathematical model of the airship flight. The model describes the dynamics, gravitation, propulsion, aerodynamics and control behavior of the airship.

2.1. Some Assumptions and Axes Systems

For the simulation, two orthogonal axes sets are supposed. First axes set is named as 'Body' axes which is pointing towards the three orthogonal X, Y and Z directions and is centered at airship's centre of volume (C.V). The second set of orthogonal axes used in the simulation is known as: 'Inertial' or 'Earth' axes system. Here X axis is aligned with the North direction, Y with the East and Z axis points downwards to the centre of Earth. For the simplicity, we assume that the body axis system coincide with the earth axes system throughout the time. The center of volume (C.V) lies on the geometrical longitudinal axes and center of gravity (C.G) on XZ plane.

It is proved from research that LTA vehicles behave much like a mass with forces and moments applied to it. The simplified format of the equation is shown below:

$$M \begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \\ \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = F_d(u, v, w, p, q, r) + A_r(u, v, w, p, q, r) + G(\lambda_{31}, \lambda_{32}, \lambda_{33}) + P \quad (1)$$

2.1.1. The Mass Matrix M

In comparison to the aircraft simulation, additional terms that are specific to buoyant vehicles are incorporated in mass matrix. These are called ‘virtual’ or ‘added’ masses and inertias. These terms arise due to the reason that vehicle mass has same order of magnitude as mass of the displaced air. Modern aerodynamic stability derivative notation is used here to denote these terms. Keeping in view the vehicle’s symmetry, mass matrix can be written:

The elements of matrix can be calculated according to the procedure outlined [10, 11].

$$M = \begin{bmatrix} m_x & 0 & 0 & 0 & m_{\dot{x}} - X_{\dot{q}} & 0 \\ 0 & m_y & 0 & -m_{\dot{x}} - Y_{\dot{p}} & 0 & m_{\dot{x}} - Y_{\dot{r}} \\ 0 & 0 & m_z & 0 & -m_{\dot{x}} - Z_{\dot{q}} & 0 \\ 0 & -m_{\dot{x}} - L_{\dot{v}} & 0 & J_x & 0 & -J_{xz} \\ m_{\dot{x}} - M_{\dot{t}} & 0 & -m_{\dot{x}} - M_{\dot{w}} & 0 & J_y & 0 \\ 0 & m_{\dot{x}} - N_{\dot{q}} & 0 & -J_{xz} & 0 & J_z \end{bmatrix} \quad (2)$$

2.1.2. The Dynamic Vector F_d

It is the 6x1 column matrix that contains the dynamic terms associated with inertial velocities u_x, v, w, p, q and r and can be expressed as:

$$F_d = [f_1 \quad f_2 \quad f_3 \quad f_4 \quad f_5 \quad f_6]^T \quad (3)$$

Its dynamic terms may be formulated in accordance with procedure proposed by [12] and [13].

2.1.3. The Aerodynamics Vector A_r :

It is a 6x1 column matrix that contains the aerodynamic forces and moments. In practice, the aerodynamic vector is written as below:

$$A_r = [A_X \quad A_Y \quad A_Z \quad A_L \quad A_M \quad A_N]^T \quad (4)$$

In body axis system, A_X, A_Y, A_Z are total aerodynamic forces on X, Y, Z axes and A_L, A_M, A_N are total aerodynamic moments about X, Y, Z axes, respectively.

The aerodynamic forces and moments can be estimated either by wind tunnel tests performed by [10, 11] or from the geometrical analytical approaches of [14, 15].

2.1.4. The Gravity and Buoyancy Vector G

It is 6x1 column matrix that contains the terms associated with gravitational and buoyancy forces and moments and is given by:

$$G = \begin{bmatrix} \lambda_{31}[W - B] \\ \lambda_{32}[W - B] \\ \lambda_{33}[W - B] \\ -\lambda_{32}a_z W \\ [\lambda_{31}a_z - \lambda_{33}a_x]W \\ \lambda_{32}a_x W \end{bmatrix} \quad (5)$$

W and B are the total weight and buoyancy force available in the vehicle. λ_{ij} are elements of direction cosine matrix which can be found by well known Quaternion technique.

2.1.5. The Propulsion Vector P

The 6x1 column matrix containing the terms associated with the propulsive forces and moments is:

$$P = [X_{prop}, Y_{prop}, Z_{prop}, L_{prop}, M_{prop}, N_{prop}]^T \quad (6)$$

where X_{prop} , Y_{prop} , Z_{prop} are total thrust along X, Y, Z axes and L_{prop} , M_{prop} , N_{prop} are total thrust moment about X, Y, Z axes, respectively.

The derivation of the mathematical expressions making each element of P depends upon propulsion system installed on the vehicle. Derivation of propulsion vector for one particular case is investigated by [16].

2.2. Linearised Equations of Motion

The model described above may be considerably simplified when it is assumed that the motion of the airship is constrained to the small perturbations about the trimmed equilibrium flight condition. Following the common practice, we assume that this motion is further divided into decoupled longitudinal and lateral motion according to [17].

2.2.1. Longitudinal Equations

Let us assume state vector for the longitudinal motion:

$$x_{long} = [u_x \quad w \quad q \quad \theta]^T \quad (7)$$

and input vector for this motion

$$u_{long} = [\delta_{er} \quad \delta_{el} \quad \delta_{tx} \quad \delta_{tz}]^T \quad (8)$$

Where elements of input vector are the right elevator deflection, left elevator deflection, thrust perturbations along X and thrust perturbations along Z axis, respectively. Then the longitudinal state equation can be written as:

$$\dot{x}_{long} = A_{long}x_{long} + B_{long}u_{long} \quad (9)$$

Simplified form of A_{long} and B_{long} matrices are given below:

$$A_{long} = \begin{bmatrix} x_{u_x} & x_w & x_q & x_\theta \\ z_{u_x} & z_w & z_q & z_\theta \\ m_{u_x} & m_w & m_q & m_\theta \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad (10)$$

$$B_{long} = \begin{bmatrix} x_{\delta_{er}} & x_{\delta_{el}} & x_{\delta_{tx}} & x_{\delta_{tz}} \\ z_{\delta_{er}} & z_{\delta_{el}} & z_{\delta_{tx}} & z_{\delta_{tz}} \\ m_{\delta_{er}} & m_{\delta_{el}} & m_{\delta_{tx}} & m_{\delta_{tz}} \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (11)$$

It shows that a small perturbation is added to the longitudinal equations of motion.

$$\dot{\theta} = q \quad (12)$$

2.2.2. Lateral Equation

Similarly, state vector for the lateral motion is:

$$x_{lat} = [v \quad p \quad r \quad \phi]^T \quad (13)$$

and input vector for lateral motion is:

$$u_{lat} = [\delta_{rt} \quad \delta_{rb} \quad \delta_{ty}]^T \quad (14)$$

Where elements of input vector are the top rudder deflection angle, bottom rudder deflection angle and thrust perturbations along Y direction, respectively. Then the lateral state equation can be written as:

$$\dot{x}_{lat} = A_{lat}x_{lat} + B_{lat}u_{lat} \quad (15)$$

Simplified form of A_{lat} and B_{lat} matrices may be written as:

$$A_{lat} = \begin{bmatrix} y_v & y_p & y_r & y_\phi \\ l_v & l_p & l_r & l_\phi \\ n_v & n_p & n_r & n_\phi \\ 0 & 1 & 0 & 0 \end{bmatrix} \quad (16)$$

$$B_{lat} = \begin{bmatrix} y_{\delta_{rt}} & y_{\delta_{rb}} & y_{\delta_{ty}} \\ l_{\delta_{rt}} & l_{\delta_{rb}} & l_{\delta_{ty}} \\ n_{\delta_{rt}} & n_{\delta_{rb}} & n_{\delta_{ty}} \\ 0 & 0 & 0 \end{bmatrix} \quad (17)$$

Similar to longitudinal equations, lateral equations of motion include additional small perturbed equation

$$\dot{\phi} = p \quad (18)$$

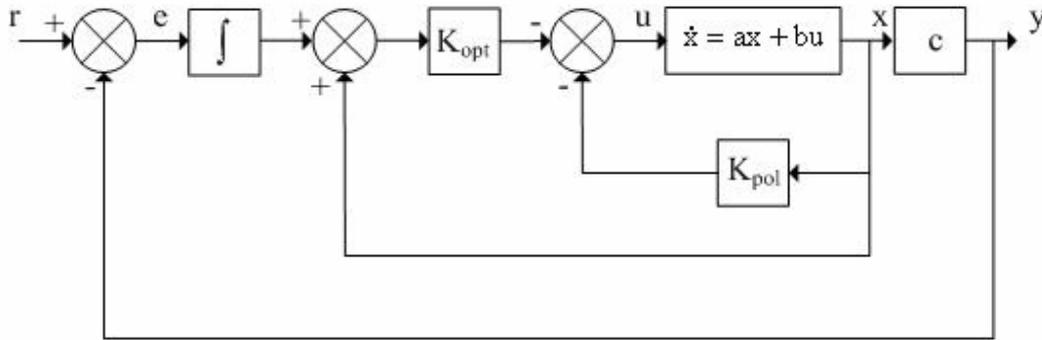


Figure 1. Airship hover controller placing specific poles, optimally controlling hover flight using integral action.

3. Control Law Design

Suppose we have the configuration for control action as shown in Fig.1. In this figure, $\dot{x} = ax + bu$ represents airship linearised state space model, with c as its output. K_{pol} and K_{opt} represent gain matrices for pole placement and optimal control gain matrices. Symbols, r , e , u , x , y , represent reference, error, input, state and output vectors in the figure, respectively. Pole placement is applied on system states at inner most loop and outer loop performs reference tracking. Optimal gains are applied on the augmented vector of states and integrated errors in this hybrid control scheme. It is to note that it is a multi input and multi output (MIMO) hybrid PI control strategy, proposed for airship 6 DOF dynamic model.

In order to derive control law for airship in the hover, first we write the feedback of the inner most loop. Actually we want some of the poles to be placed at its more stable region which are producing oscillations in the system dynamics. These poles, due to their presence near the stability boundary, exhibit oscillatory behavior only at its hover condition indicated by Gomes and Cook. As the state feedback is possible according to [16], we place these poles to the desired stable position. The modified system matrix after inner most feedback loop is:

$$a^* = a - bK_{pol} \quad (19)$$

where a^* is the modified system matrix.

For the purpose of hover control, we can design optimal hover controller with integral action using outer feedback loop in the light of theory explained by [19-21]. If dynamic system has output equation,

$$y = Cx \quad (20)$$

We can find optimal hover controller $u = \Pi(e, x)$ minimizing the performance function.

The error is expressed as:

$$e = r_i - y \quad (21)$$

where r_i is the reference input.

For the MIMO systems, we have

$$e = nr_i - y = nr_i - Cx \quad (22)$$

$$e = \begin{bmatrix} n_{11} & 0 & \dots & 0 & 0 \\ 0 & n_{22} & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & n_{b-1b-1} & 0 \\ 0 & 0 & \dots & 0 & n_{bb} \end{bmatrix} \begin{bmatrix} r_{i1} \\ r_{i2} \\ \vdots \\ r_{ib-1} \\ r_{ib} \end{bmatrix} +$$

$$\begin{bmatrix} c_{11} & c_{12} & \dots & c_{1n-1} & c_{1n} \\ c_{21} & c_{22} & \dots & c_{2n-1} & c_{2n} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ c_{b-11} & c_{b-12} & \dots & c_{b-1n-1} & c_{b-1n} \\ c_{b1} & c_{b2} & \dots & c_{bn-1} & c_{bn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{n-1} \\ x_n \end{bmatrix}$$

If n is number of states, b is number of reference inputs and m is the number of inputs of the dynamic system, then $n \in \mathfrak{R}^{b \times b}$ and $c \in \mathfrak{R}^{b \times n}$.

Using the output equation and denoting:

$$\begin{aligned} e &= \dot{x}_{ref} , \\ \dot{x}_{ref} &= nr_i - y = nr - cx \end{aligned} \quad (23)$$

If we augment the system dynamics $\dot{x} = \bar{a}x + bu$ with this equation, we have:

$$\begin{aligned} \dot{x} &= a^* x + bu, & y &= cx \\ \dot{x}_{ref} &= nr_i - y = nr_i - cx . \end{aligned}$$

That is,

$$X = AX + Bu + \bar{n}r_i , \quad y = cx . \quad (24)$$

Where,

$$X = \begin{bmatrix} x \\ \dot{x}_{ref} \end{bmatrix} \in \mathfrak{R}^c \quad (c = n + b)$$

is the augmented state vector,

$$\begin{aligned} A &= \begin{bmatrix} a^* & 0 \\ -c & 0 \end{bmatrix} \in \mathfrak{R}^{c \times c} \\ B &= \begin{bmatrix} b \\ 0 \end{bmatrix} \in \mathfrak{R}^{c \times m} \\ \bar{n} &= \begin{bmatrix} 0 \\ n \end{bmatrix} \in \mathfrak{R}^{c \times b} \end{aligned}$$

are the time invariant matrices of coefficients.

Quadratic performance function is as:

$$J \left(\begin{bmatrix} x(\cdot) \\ x_{ref}(\cdot) \end{bmatrix}, u(\cdot) \right) = \frac{1}{2} \int_{t_0}^{t_f} \left(\begin{bmatrix} x \\ x_{ref} \end{bmatrix}^T Q \begin{bmatrix} x \\ x_{ref} \end{bmatrix} + u^T R u \right) dt \quad (25)$$

Where, $Q \in \mathfrak{R}^{c \times c}$ is positive-semi definite constant coefficient matrix and $R \in \mathfrak{R}^{m \times m}$ is positive definite constant coefficient matrix.

By using the quadratic performance function, one obtains the Hamiltonian function:

$$H \left(X, u, r_i, \frac{\partial V}{\partial X} \right) = \frac{1}{2} (X^T Q X + u^T R u) + \left(\frac{\partial V}{\partial X} \right)^T \times (AX + Bu + \bar{n}r_i) \quad (26)$$

From this, we have:

$$\frac{\partial H \left(X, u, r_i, \frac{\partial V}{\partial X} \right)}{\partial u} = u^T R + \left(\frac{\partial V}{\partial X} \right)^T B$$

One can find the control law using the first order necessary condition for optimality.

We then have the following form of an optimal control algorithm:

$$u = -R^{-1} B^T \frac{\partial V(X)}{\partial X} = -R^{-1} \begin{bmatrix} b \\ 0 \end{bmatrix}^T \frac{\partial V \left(\begin{bmatrix} x \\ x_{ref} \end{bmatrix} \right)}{\partial \begin{bmatrix} x \\ x_{ref} \end{bmatrix}} . \quad (27)$$

The solution of Hamilton-Jacobi-Bellman partial differential equation

$$-\frac{\partial V}{\partial t} = \frac{1}{2} X^T Q X + \left(\frac{\partial V}{\partial X} \right)^T A X - \frac{1}{2} \left(\frac{\partial V}{\partial X} \right)^T B R^{-1} B^T \frac{\partial V}{\partial X} \quad (28)$$

is satisfied by quadratic return function

$$V(X) = \frac{1}{2} X^T K_{opt} X . \quad (29)$$

$K_{opt} \in \mathfrak{R}^{c \times c}$ is symmetric matrix.

$$- \dot{K}_{opt} = Q + A^T K_{opt} + K_{opt} A - K_{opt} B R^{-1} B^T K_{opt} , \quad K(t_f) = K_{f_{opt}} . \quad (30)$$

which gives the unknown symmetric matrix K_{opt} .

The above equation needs to be solved.

Controller is found using the equations (27) and (29) as:

$$u = -R^{-1} B^T K_{opt} X = -R^{-1} \begin{bmatrix} b \\ 0 \end{bmatrix}^T K_{opt} \begin{bmatrix} x \\ x_{ref} \end{bmatrix} . \quad (31)$$

From $\dot{x}_{ref} = e$, we can have, $x_{ref} = \int e dt$

we have optimal control law with integral action as:

Thus the control law for the hovering control of an airship can be written as:

$$u = -R^{-1} \begin{bmatrix} b \\ 0 \end{bmatrix}^T K_{opt} \begin{bmatrix} x \\ \int e dt \end{bmatrix} \quad (32)$$

4. Hovering Control of an Airship

We begin our investigations with the state space equation:

$$\dot{x} = ax + bu, \quad x(t_0) = x_0$$

The states for this system are:

$$x = [u_x \quad w \quad q \quad \theta \quad v \quad p \quad r \quad \phi \quad \varphi \quad N \quad E \quad h]^T$$

Where, u_x, v, w are the linear velocities along X, Y, Z axes p, q, r are the angular velocities around X, Y, Z axes, ϕ, θ, φ are attitude angles about X, Y, Z axes and N, E, h are the position variables in earth axes system which is supposed to be coincide with XYZ axes system.

Control input is,

$$u = [\delta_{er} \quad \delta_{el} \quad \delta_{tx} \quad \delta_{tz} \quad \delta_{rt} \quad \delta_{rb} \quad \delta_{ty}]^T$$

Where, elements are right elevator deflection, left elevator deflection, thrust along X axis, thrust along Z axis, top rudder deflection, bottom rudder deflection and thrust along Y direction, respectively.

It is evident from the work presented by Cook that the linearised model of airship is approximately similar to its nonlinear one. So we can use Longitudinal-Lateral dynamics to represent its state-space model indicated by [18].

$$\dot{x} = ax + bu$$

where,

$$a = \begin{bmatrix} x_{u_x} & x_w & x_q & x_\theta & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ z_{u_x} & z_w & z_q & z_\theta & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ m_{u_x} & m_w & m_q & m_\theta & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & y_v & y_p & y_r & y_\phi & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & l_v & l_p & l_r & l_\phi & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & n_v & n_p & n_r & n_\phi & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$b = \begin{bmatrix} x_{\delta_{er}} & x_{\delta_{el}} & x_{\delta_{tx}} & x_{\delta_{tz}} & 0 & 0 & 0 \\ z_{\delta_{er}} & z_{\delta_{el}} & z_{\delta_{tx}} & z_{\delta_{tz}} & 0 & 0 & 0 \\ m_{\delta_{er}} & m_{\delta_{el}} & m_{\delta_{tx}} & m_{\delta_{tz}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & y_{\delta_{rt}} & y_{\delta_{rb}} & y_{\delta_{ty}} \\ 0 & 0 & 0 & 0 & l_{\delta_{rt}} & l_{\delta_{rb}} & l_{\delta_{ty}} \\ 0 & 0 & 0 & 0 & n_{\delta_{rt}} & n_{\delta_{rb}} & n_{\delta_{ty}} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

It is to noted that we have included additional approximated dynamic equations in the above formulation :

$$\dot{\phi} = r, \quad \dot{N} = u_x, \quad \dot{E} = v, \quad \dot{h} = w$$

By proper choice of $K_{f_{pol}}$, we can place some of the marginally stable poles to the desired stable region: $a^* = a - bK_{f_{pol}}$

where, a^* is the modified system matrix.

For the implementation of optimal hover control with integral action, error vector is expressed in terms of reference inputs and output equation as: $e = r_i - y$, where:

$$r_i = \begin{bmatrix} r_\theta \\ r_\phi \\ r_\varphi \\ r_x \\ r_y \\ r_z \end{bmatrix} \text{ and } y = cx = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_x \\ w \\ q \\ \theta \\ v \\ p \\ r \\ \phi \\ \varphi \\ N \\ E \\ h \end{bmatrix}$$

$$X = [u_x \quad w \quad q \quad \theta \quad v \quad p \quad r \quad \phi \quad \varphi \quad N \quad E \dots \\ h \quad \dot{x}_{ref_\theta} \quad \dot{x}_{ref_\phi} \quad \dot{x}_{ref_\varphi} \quad \dot{x}_{ref_x} \quad \dot{x}_{ref_y} \quad \dot{x}_{ref_z}]^T$$

$$A = \begin{bmatrix} a^* & 0 \\ -c & 0 \end{bmatrix}, B = \begin{bmatrix} b \\ 0 \end{bmatrix}, \bar{n} = \begin{bmatrix} 0 \\ n \end{bmatrix}$$

Minimization of quadratic performance function gives the above mentioned controller:

$$u = -R^{-1} \begin{bmatrix} b \\ 0 \end{bmatrix}^T K_{opt} \begin{bmatrix} x \\ \int edt \end{bmatrix}$$

By using modified airship dynamic,

$$\dot{x} = a^* x + bu$$

and

$$\dot{x}_{ref} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_\theta \\ r_\phi \\ r_\varphi \\ r_x \\ r_y \\ r_z \end{bmatrix} - cx$$

or

$$\begin{bmatrix} \dot{x}_{ref_\theta} \\ \dot{x}_{ref_\phi} \\ \dot{x}_{ref_\varphi} \\ \dot{x}_{ref_x} \\ \dot{x}_{ref_y} \\ \dot{x}_{ref_z} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_\theta \\ r_\phi \\ r_\varphi \\ r_x \\ r_y \\ r_z \end{bmatrix} -$$

$$\begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u \\ w \\ q \\ \theta \\ v \\ p \\ r \\ \phi \\ \varphi \\ N \\ E \\ h \end{bmatrix}$$

We get an augmented model:

$$X = AX + Bu + \bar{n}r_i, \quad y = cx, \\ X_0(t_0) = X_0$$

where,

5. Simulation and Results

5.1. Simulation Strategy and Airship Parameters

The initial and final reference values of the $u_x, v, w, p, q, r, \phi, \theta, \varphi, x, y, z$ are given at the start of simulation using the basics stated by [21] and [22]. Body axes system is assumed to coincide with Earth axes system all the time.

5.2. Example Airship Parameters Selected:

- Total Mass = 41500 Kg
- Length = 100 m
- Max Diameter = 25 m
- 4 control surfaces (2 Rudders & 2 Elevators) in "+" fashion.
- 3 Propellers (In the X, Y and Z directions).

5.3. Simulation Time

Total simulation period selected in this work is equal to 150 seconds.

5.4. Simulation Tool: MATLAB 7

5.5. Simulation Results

No Wind Case

First the system is tested in hover flight with no wind (suppose at no wind, wind speed = 0.2m/s). Airship starts from the initial position, $[\theta \quad \phi \quad \varphi \quad N \quad E \quad h]^T = [10 \quad 10 \quad 10 \quad 10 \quad 10 \quad -10]^T m$ and becomes stationary at its given reference position, $[\theta \quad \phi \quad \varphi \quad N \quad E \quad h]^T = [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad -10]^T m$

The purpose of the simulation is to show the ability of the controller to sustain the reference

position for the airship. Simulation results are shown in the Fig.2. It is clear from the results that the proposed controller takes the airship to a stable position in almost 35 seconds, achieving the airship hovering flight successfully. It can also be seen that the initial transitions in the position and attitude variables are due to the presence of aerodynamic vector in the dynamic model of airship.

5.6. Light Wind Case

System is tested in a very light constant wind (1.0m/s) flowing opposite to the X direction. Airship starts from the initial position, $[\theta \ \phi \ \varphi \ N \ E \ h]^T = [10 \ 10 \ 10 \ 10 \ 10 \ -10]^T m$ and becomes stationary at the given reference position,

$$[\theta \ \phi \ \varphi \ N \ E \ h]^T = [0 \ 0 \ 0 \ 0 \ 0 \ -10]^T m.$$

Simulation results are shown in the Fig.3. It is evident from the figures that the proposed controller takes the airship to a stable position in almost 30 seconds, sustaining the airship hovering flight. Increased action of the aerodynamic forces can be seen in the initial part of the response as compared to the no wind case. Improvement in the settling time is due to the increased effectiveness of the control surfaces like rudders and elevators.

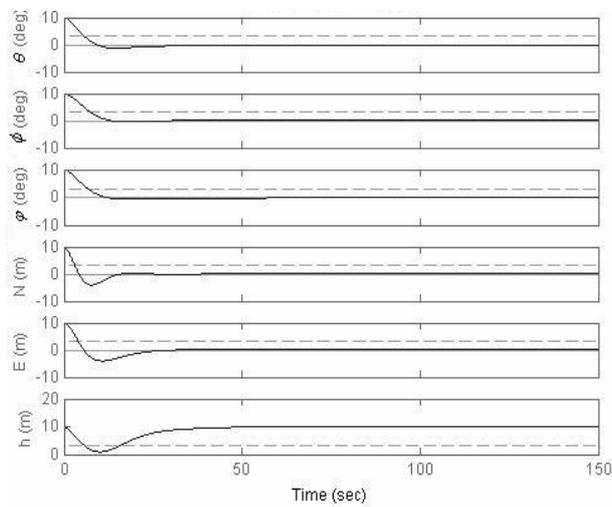


Figure 2. Airship attitude and position variables ($\theta, \phi, \varphi, N, E, h$) in hovering flight at no wind case for MIMO Hybrid PI control scheme.

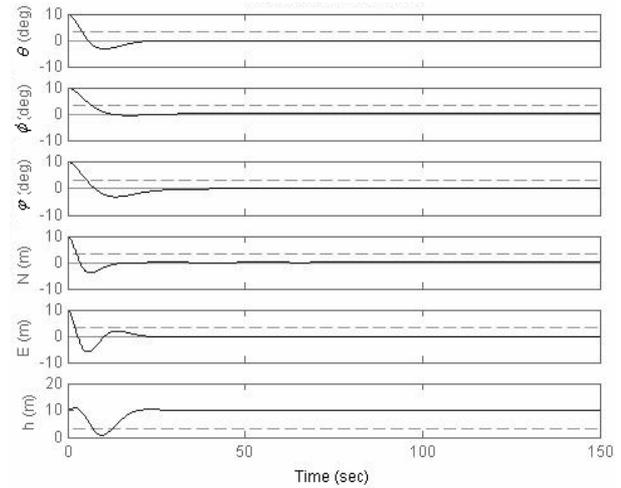


Figure 3. Airship attitude and position variables ($\theta, \phi, \varphi, N, E, h$) in hovering at light wind case.

5.7. Normal Wind Case1

System is tested in a normal constant wind (3.0m/s) flowing opposite in the X direction. Airship starts from the initial position, $[\theta \ \phi \ \varphi \ N \ E \ h]^T = [10 \ 10 \ 10 \ 10 \ 10 \ -10]^T m$ and becomes stationary at the reference position, $[\theta \ \phi \ \varphi \ N \ E \ h]^T = [0 \ 0 \ 0 \ 0 \ 0 \ -10]^T m$.

Simulation results are shown in the Fig. 4. It can be seen from the figures that the controller takes the airship to a stationary position in normal wind case within 25 seconds. This is the usual wind scenario facing the airship. It is clear from the response that the suggested controller achieves its goal successfully

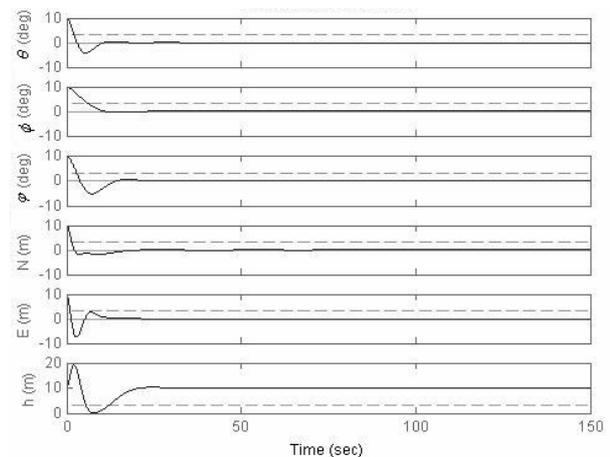


Figure 4. Airship attitude and position variables ($\theta, \phi, \varphi, N, E, h$) in hovering flight at normal wind case.

5.8. Normal Wind Case2:

System is tested in a normal constant wind (3.0m/s) flowing opposite to the X direction. Airship starts from the different initial position, $[\theta \ \phi \ \varphi \ N \ E \ h]^T = [20 \ 20 \ 20 \ -10 \ -10 \ -20]^T m$ and becomes stationary at its given reference position,

$$[\theta \ \phi \ \varphi \ N \ E \ h]^T = [0 \ 0 \ 0 \ 0 \ 0 \ -10]^T m.$$

Simulation results are shown in the Fig.5. It is evident from the simulation results that the suggested controller takes the airship to a pre defined stationary position even starting from different initial points, which shows the effectiveness of the control design.

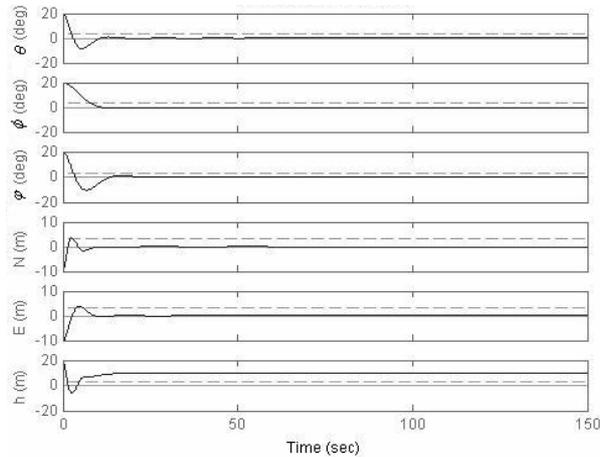


Figure 5. Airship attitude and position variables ($\theta, \varphi, \phi, N, E, h$) in hovering flight at normal wind with different starting point .

5.9. High Wind with Turbulence Case

System is tested in a high constant wind (8.0m/s) flowing opposite to the X direction with turbulence in all plant inputs. Turbulence is supposed to be a Gaussian noise having standard deviation equal to 20. This is the maximum disturbance estimated in any state [9]. Airship starts from the initial position, $[\theta \ \phi \ \varphi \ N \ E \ h]^T = [20 \ 20 \ 20 \ -10 \ -10 \ -20]^T m$ and becomes stationary at the reference position, $[\theta \ \phi \ \varphi \ N \ E \ h]^T = [0 \ 0 \ 0 \ 0 \ 0 \ -10]^T m$.

Simulation results are shown in the Fig.6. We can see from the figures that the hover controller tracks efficiently its reference values even in the presence of large disturbance noise in all control inputs, simultaneously. This explains effectiveness and robustness in the proposed control design.

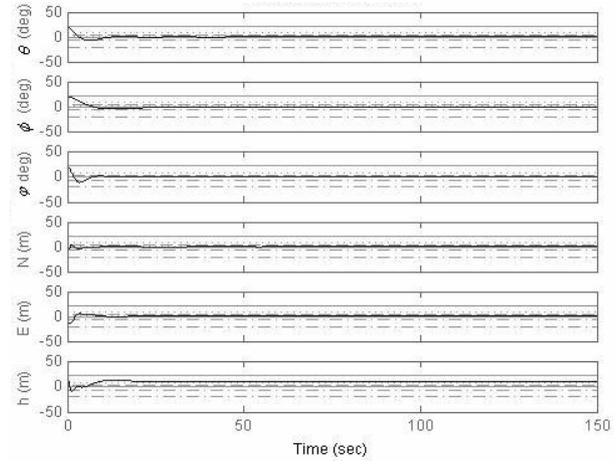


Figure 6. Airship attitude and position variables ($\theta, \varphi, \phi, N, E, h$) in hovering flight at high wind with disturbance input case.

5.10. Controllability, Stabilizability, Robustness and Comparison

Its controllability matrix can be checked by,

$$C_1 = [b \ ab \ a^2b \ a^3b \ \dots \ a^{17}b]$$

$$\text{Rank}(C_1) = 12.$$

System is completely controllable.

Similarly, for the augmented system, the controllability by,

$$C_2 = [B \ AB \ A^2B \ A^3B \ \dots \ A^{17}B]$$

$$\text{Rank}(C_2) = 18.$$

It is completely controllable.

Eigen values of optimal system in normal wind are as:

- 1.0571, -0.5297 + 0.9076i, -0.5297 - 0.9076i, -0.8928, -0.4483 + 0.7609i, -0.4483 - 0.7609i,
- 0.5248, -0.2723 + 0.4316i, -0.2723 - 0.4316i, -0.4016, -0.2230 + 0.3338i, -0.2230 - 0.3338i,
- 0.2781, -0.1968 + 0.2283i, -0.1968 - 0.2283i, -0.2287 + 0.1664i, -0.2287 - 0.1664i, -0.0015.

In this system it can be seen that there exist the state feedback law and all poles are in the left half plane, so the system is stabilizable.

As this system is controllable and stable, it can be viewed that no zero occur at $s = 0$ that may cancel the integrator which has transfer function equal to $1/s$, so it can be said that in this configuration, the output y will track asymptotically any step reference input even with the presence of disturbance or plant parameter variations, thus satisfying the robustness in the hover control scheme suggested by [19].

5.11. Comparison of Results

In the control scheme presented by [9], attitude and position stabilization occur in 10-50 seconds in no wind scenario. In normal wind (3.0m/s), stabilization in saturated case occurs in 35-150 seconds. These results are taken neglecting the aerodynamic vector from the model. In our case, attitude and position stabilization occur in about 10-40 sec in all wind cases without excluding the aerodynamic vector from the model. This can be considered as improvement in the control design. Stabilization time may be further improved with more proper choice of feedback and Q, R matrices.

Similarly for the control scheme presented by [8], although aerodynamic vector is included but the control action performed is less smooth as compared with the design suggested in this paper.

6. Conclusions

Hovering flight of airship can be used for observation oriented applications. To maintain airship hovering capability, a simple but efficient control scheme is required which can be easily analyzed, modified with the help of some software tools like MATLAB. In the proposed control scheme, we have used the principles of linearity to design a control law that satisfies its robustness by rejecting any disturbance input in the plant. The hybrid control scheme designed here uses the integration action, state feedback and optimal gains which may be easily implemented with the help of analog and digital computers. Simulation results and their comparison can be viewed as improvement in the proposed control design. Results can be tuned further by more proper choice of feedback matrices. The suggested scheme shows that it is a simple and effective design for the airship hover control applications.

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